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Transmission of quantum states through disordered channels with dimerized defects

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Abstract
We study a quantum-state transfer protocol between two end spins of a disordered spin-1/2 chain. We particularly evaluate the performance of the channel when it is subjected to short-range correlated on-site fluctuations. This is implemented by generating a random distribution of dimer-like defects across the chain featuring a given on-site potential W. By numerically evaluating the system's natural time evolution, we report the possibility of carrying out high-fidelity quantum-state transmission from one end to another given their local frequency is set $\varepsilon \sim W$ and W is within the energy band of the defectless chain.

Keywords Quantum-state transfer · Anderson localization · Correlated disorder

1 Introduction

The dynamics of a single quantum particle in disordered media has been a very active field of research, since Anderson's seminal paper [1], further culminating in the so-called phenomenon of Anderson localization, now spanning through a wide range of areas and backed up by strong technological and theoretical achievements [2]. Basically, the model consists of an electron hopping among nearest-neighbor sites of a lattice displaying on-site random potentials, these accounting for the local interaction between the electron and the atoms in an amorphous material.

In low-dimensional systems ($d \leq 2$), all eigenstates become exponentially localized for any finite amount of disorder leading to the absence of diffusion and for $d = 3$, there is a metal–insulator phase transition coined as Anderson transition [2–4]. Anderson localization theory came to have an astonishing impact on, e.g., acoustics [5] light

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propagation [6], photonic bandgap materials [7], cold atoms [8], and many other physical platforms (see [2] for a review).

Several years ago, it was shown that the Anderson localization theory in low-dimensional disordered systems can be violated when affected by *correlated* fluctuations [9–14] (cf. [15,16] for experimental realizations). Very recently, the existence of a single-particle mobility edge was reported in a 1D optical lattice, originating from a finite correlation length in the speckle potential [17]). Correlated disorder may thus be present depending on the experimental setting at hand, or be judisciouly configured [15], in various forms [18]. One of the precursors of this line of investigation was the random dimer model put forward in Ref. [10]. The framework consists of a chain with two uncorrelated random on-site energies ε_a and ε_b. It was reported that whenever one or both of these defects occur in pairs, that is side-by-side, an extended state can be found within the band of allowed energies [10] . In Ref. [11], it was shown that some conducting polymers can actually be mapped to a random dimer model. Moreover, from the experimental point of view, the effect of random dimer correlations in GaAs-AlGaAs superlattices was investigated in [12]. There it was confirmed that dimer-like correlations in a disordered semiconductor indeed entails the appearance of extended states, thus corroborating with previous results [10,11].

The interplay of localization and delocalization in the framework of chains with correlated disorder also finds appeal in the context of distributed quantum information processing, as shown recently in [19]. In general, in order to transfer quantum states and create entanglement between different processing units of a quantum network one needs to establish reliable quantum communication channels between the nodes [20]. In this context, much attention has been given to spin chains with engineered couplings that could act as quantum buses for short- and mid-range quantum communication protocols [19,21–34] (for reviews see, e.g., [35] and [36]) These have the advantage of a lower degree of external control and dismiss the need of interconverting between photonic and atomic degrees of freedom [37,38].

A handful of schemes for quantum-state transfer (QST) tasks has been proposed since the overall concept was put forward in [21]. In [22,39,40] it was shown that perfect QST is possible in fully engineered chains assuring a linear dispersion law. The downside of that kind of setting is the requirement to judiciously tuning the entire chain. In [28,29] an alternative method was proposed which involves optimizing only the outermost couplings of the chain in order to perform QST in the ballistic regime. Other configurations are based on energetically detaching the communicating parties from the rest of the chain so that they span their own subspace. That can be done, e.g., by applying suitable magnetic fields locally [30,32,41] or linking the sender and receiver very weakly to the channel [24–26,33,42,43]. These weak-coupling models offer great resilience against noise and low engineering requirements with the expense of having long transfer times. Taking δ as the perturbative parameter, the transfer time scales $\sim O(\delta^{-1})$ or $\sim O(\delta^{-2})$ depending on resonance conditions between the sender/receiver and the channel [25]. Despite that, there is also the possibility of designing decoherence-free subpaces by weakly coupling two registers through a noisy channel [44]. Chains with weak end bonds also find use in generating long-distance entanglement [45,46] and recently have been realized experimentally in a bulk material [47] (see also [48] for a study toward implementing it using superconducting circuits).

In general, when designing solid-state devices for transmitting quantum information one should bear in mind the unavoidable presence of experimental noise, such as static imperfections coming from the manufacturing process, thermal fluctuations, etc. For this very reason, the reliability of many spin channels against disorder have been widely investigated [19,49–56]. While most of schemes are able to hold against small amounts of it, there is also the prospect of using it to modify, on demand, the performance of the channel [19,57,58]. In [19], for instance, it was put forward the idea of using spatially long-range correlated disorder in order to control entanglement distribution patterns in XY chains.

In this work, we will investigate the possibility of carrying out a QST protocol [21] across a disordered channel [53] with *short-range* correlations. For such, we rely on the weak-coupling spin model [24–26], where both communicating parties are very weakly connected to each end of the chain. We introduce correlations in the channel upon randomly generating pairs of on-site defects located right next to each other all having the same energy. We show that high-fidelity QST can be set to occur when the energy of the outer spins matches the one associated with the defects, this being in contrast with the uncorrelated scenario—that is with the channel featuring randomly distributed *single*-site defects—where strongly localized states take over the entire spectrum of the channel thereby spoiling the transmission.

2 Model and formalism

Let us consider a 1D chain with $N+2$ spins 1/2 interacting through a XY Hamiltonian of the form ($\hbar = 1$)

$$\hat{H} = \sum_{i=1}^{N+2} \frac{\omega_i}{2}(\hat{1} - \hat{\sigma}_i^z) + \sum_{i=1}^{N+1} \frac{J_i}{2}\left(\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \hat{\sigma}_i^y\hat{\sigma}_{i+1}^y\right), \tag{1}$$

where $\hat{\sigma}_i^{x,y,z}$ are Pauli spin operators for the spin located at site i, ω_i is the local potential, and J_i denotes the spin coupling strength.

In the usual QST protocol [21], Alice controls the first spin of the chain and prepares an arbitrary qubit state $|\phi\rangle = a|0\rangle + b|1\rangle$ aimed to be sent down the (initially polarized) channel to be further retrieved by Bob at some other location, here taken to be the last site of the chain. The state of the whole system is thus initialized in $|\Psi(0)\rangle = |\phi\rangle|0\rangle \ldots |0\rangle$. Given the above Hamiltonian preserves the number of excitations, the relevant dynamics will only take place on the single-excitation subspace spanned by $|j\rangle \equiv \sigma_j^+|\emptyset\rangle$, σ^+ being the spin raising operator (state $|\emptyset\rangle \equiv |0\rangle \ldots |0\rangle$ is stationary).

Setting $\omega_1 = \omega_{N+2} = \varepsilon$, $J_1 = J_{N+1} = g$, and $J_i = J$ elsewhere, we conveniently rewrite the system's Hamiltonian as

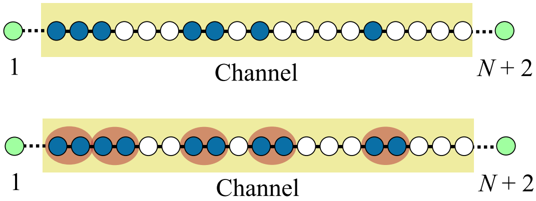


Fig. 1 Schematic view of the chain. Both end spins (at sites 1 and $N+2$) with local energy ε are weakly connected to the channel with strength g (dashed lines). The couplings within the channel (solid lines) are all uniform (J), but diagonal disorder is present in the form of local **a** uncorrelated single site or **b** correlated dimer-like defects with energy W (filled circles) occurring with probability p. Defect segments always comprises an even number of sites for dimerized correlations

$$H = \varepsilon(|1\rangle\langle 1| + |N+2\rangle\langle N+2|) + \sum_{j=2}^{N+1} \omega_j |j\rangle\langle j| + g(|1\rangle\langle 2| + |N+1\rangle\langle N+2|) + J\sum_{j=2}^{N} |j\rangle\langle j+1| + \text{H.c.} \tag{2}$$

In summary, both sender and receiver, sites 1 and $N+2$, have local tunable energy ε and are coupled to the bulk through $g \ll J$ (weak-coupling regime [24]). Herein, we take $J \equiv 1$ as the energy unit. Now, diagonal disorder is introduced in the channel in the following way. We initially set $\omega_j = 0$, with $j = 2, 3, \ldots, N+1$. For uncorrelated random defects (Fig. 1a), we attribute to *each* site a probability p for gaining energy of strength $\omega_j = W$, leaving $\omega_j = 0$ otherwise. For *correlated* dimer-like defects (Fig. 1b), instead, we set $\omega_j = W$ *and* $\omega_{j+1} = W$ with a given probability p and keep running this coin-flip procedure from the next available site with zero energy, and so forth, until site N.

The figure of merit of the QST protocol from site 1 to $N+2$ can be evaluated using the input-averaged fidelity [21]

$$F(t) = \frac{1}{2} + \frac{f_{N+2}(t)}{3}\cos\vartheta + \frac{f_{N+2}(t)^2}{6}, \tag{3}$$

where $f_{N+2}(t) = |\langle N+2|\mathcal{U}(t)|1\rangle|$ is the absolute value of transition amplitude, with $\mathcal{U}(t) \equiv e^{-iHt}$ being the quantum time evolution operator, and ϑ is the amplitude phase, which generally can be ignored by a convenient choice of the of the on-site potentials.

In weak-coupling models, one can either look for a convenient, end-to-end symmetric channel eigenstate to *mediate* the transmission and place the outer spins in narrow resonance with it or the transfer may take place via an effective Rabi dynamics between the outer spins, when they are off-resonantly coupled to the channel [25,33]. Here, we address the latter case as there will be no fixed channel eigenvalues to tune with due to disorder. In that scenario, it is expected that the dynamics involving spins 1 and $N+2$ reduces to that of a two-level system when g is perturbatively small. Ideally, one should get a couple of eigenstates of the form $|\psi^{\pm}\rangle \simeq (|1\rangle \pm |N+2\rangle)/\sqrt{2}$

thus leading to an almost perfect QST in a time that goes $\sim \Omega^{-1}$, with Ω being the characteristic Rabi splitting [33].

Indeed, using a a second-order perturbation-theory approach in g, it is possible to obtain an effective Hamiltonian that accounts for the renormalized coupling between the outer spins in terms of the normal modes of the *channel* only—that is, considering sites 2 through $N+1$ in Eq. (1)—, say $|E_k\rangle = \sum_{j=2}^{N+1} a_{j,k}|j\rangle$ with corresponding energies E_k. It reads (see [25] for details)

$$\hat{H}_{\text{eff}} = \begin{pmatrix} h_L & J_{\text{eff}} \\ J_{\text{eff}} & h_R \end{pmatrix}, \tag{4}$$

with

$$h_\nu = \varepsilon - g^2 \sum_k \frac{|a_{\nu,k}|^2}{E_k - \varepsilon}, \tag{5}$$

$$J_{\text{eff}} = -g^2 \sum_k \left(\frac{a_{L,k} a_{R,k}^*}{E_k - \varepsilon} \right), \tag{6}$$

where $\nu \in \{L, R\}$, $a_{L,k} \equiv \langle 2|E_k\rangle$, and $a_{R,k} \equiv \langle N+1|E_k\rangle$.

3 Results and discussion

A careful look into Eqs. (4) and (5) tells us that if, eventually, $\{|E_k\rangle\}$ contains localized-like states, their underlying asymmetry entails $|a_{L,k}| \neq |a_{R,k}|$ and so $\Delta \equiv h_L - h_R \neq 0$ (in the absence of particle-hole symmetry), i.e., it induces an effective on-site impurity in Hamiltonian (4), thereby suppressing the effective Rabi dynamics between sites 1 to $N+2$ and thus the transfer fidelity [43]. For strong amounts of (uncorrelated) disorder, the qubit would remain trapped at the sender's site. On the other hand, some correlated noise distributions can breakdown (or suppress) localization in some parts of the energy band [10,11,13,19]. Therefore tuning ε into those zones may lead to $|\Delta/J_{\text{eff}}| \ll 1$, owing to the presence of delocalized-like modes.

We now turn our attention to the localization properties of the channel with short-range correlated disorder in the form of random dimerized defects having energy W to check whether and where it holds localization-free zones in the spectrum. This can be done by evaluating the participation ratio of eigenstates

$$P(E_k) = \frac{1}{N \sum_{j=2}^{N+1} |a_{j,k}|^4}, \tag{7}$$

giving $P(E_k) = 1/N$ for a fully localized state and $P(E_k) = 1$ for an extended state with equal coefficients, i.e., $a_{j,k} = 1/\sqrt{N}$ for every j. For an extended harmonic Bloch state with $a_{j,k} \sim \sin[\pi k j/(N+1)]$ the participation reaches $P(E_k) = 2/3$. By carrying out exact numerical diagonalization of Hamiltonian (2) without spins 1 and $N+2$, in Fig. 2 we plot the participation ratio against E_k (in units of J) and

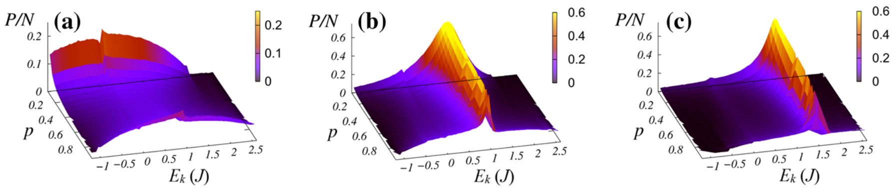


Fig. 2 Participation ratio of channel's eigenstates, $P(E_k)$, versus E_k and p for **a** uncorrelated random defects with energy $W/J = 1$ and (**b** and **c**) correlated random dimers with $W/J = 1$ and 1.5, respectively. Each $P(E_k)$ was averaged over 10^2 independent realizations of disorder for $N = 500$. Note that correlated disorder allows for extended Bloch-like states around $E_k \approx W$ with $P(E_k) \approx 2/3$

the local probability p of generating a defect for uncorrelated (Fig. 2a) and correlated (dimerized) disorder (Fig. 2b and c). In the former case, we note that $P(E_k)$ is small (much less than 2/3) thereby suggesting the occurrence of Anderson localization [10,11]. For correlated disorder, shown for $W/J = 1$ and 1.5 in Fig. 2b and c, respectively, we check that $P/N \approx 2/3$ when $E_k \approx W$. The mode with $E_k = W$ is an extended harmonic Bloch-like state with random phase changes at the dimer defects. This resonant mode is present whenever the defect energy W is within the band of allowed energies of the defectless chain (in the present case for $W \leq 2J$). This feature suggests that a few modes lying around level W remains with an extended profile despite disorder in finite chains, in agreement with previous works over that class of defects [10,11]. That is the very ingredient required to achieve an efficient quantum state transfer.

Given the pair of eigenstates $|\psi^{\pm}\rangle$ of the effective Hamiltonian, Eq. (4), the transition amplitude reads

$$f_{N+2}(t) = \Lambda \left| \sin\left(\frac{\Omega t}{2}\right) \right|, \tag{8}$$

where $\Lambda \equiv 2|\langle N+2|\psi^{\pm}\rangle\langle\psi^{\pm}|1\rangle| = 2|J_{\text{eff}}|/\Omega$ is the end-to-end correlation amplitude and $\Omega = \sqrt{\Delta^2 + 4J_{\text{eff}}^2} \sim O(g^2)$. The transition amplitude reaches its maxima, $f_{N+2}(\tau) = \Lambda$, at times $\tau = n\pi/\Omega$, with $n = 1, 3, \ldots$ and $f_{N+2}(\tau) \simeq 1$ [and thus $F(\tau) \simeq 1$; see Eq. (3)] only when $|\Delta| \ll |J_{\text{eff}}|$. Evaluating Λ itself then becomes a practical way for accessing the capability of the channel to support a high-fidelity QST protocol.

In Fig. 3, we plot Λ (averaged over several samples) against the sender/receiver tuning energy ε for many disorder strengths W for $N = 50$. For correlated, dimerized defects (solid curves), whenever ε gets close to W, the end-to-end correlation amplitude reaches about its maxima $\Lambda \approx 1$. This optimal transfer resonance gets incredibly sharp as W is further increased, which brings Λ into gradual declining. Nevertheless, it is rather appealing the fact that Λ is still very close to 1 for strong amounts of disorder, say, $W = 4J$. For uncorrelated random single defects (dotted curves), however, Λ barely develops over 1/2 already for $W = 1$. For stronger W, the possibility of QST is completely ruled out.

Another interesting feature to look after is how sensitive Λ is to the size of the channel N. By fixing $\varepsilon = W$ (namely, the optimal resonance spot as depicted previously in Fig. 3), in Fig. 4, we plot Λ against W for numerous sizes ranging from $N = 30$ to

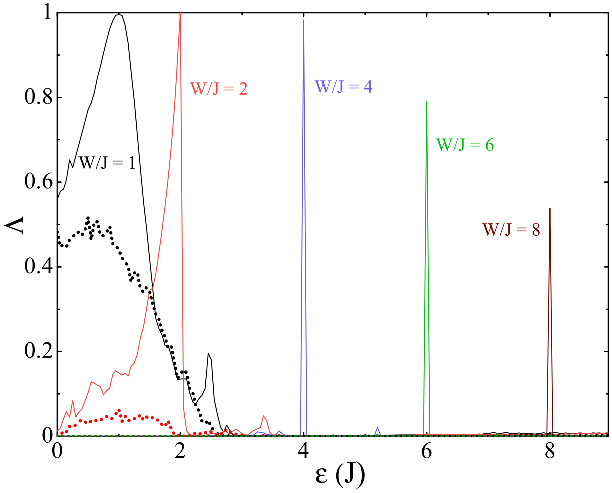
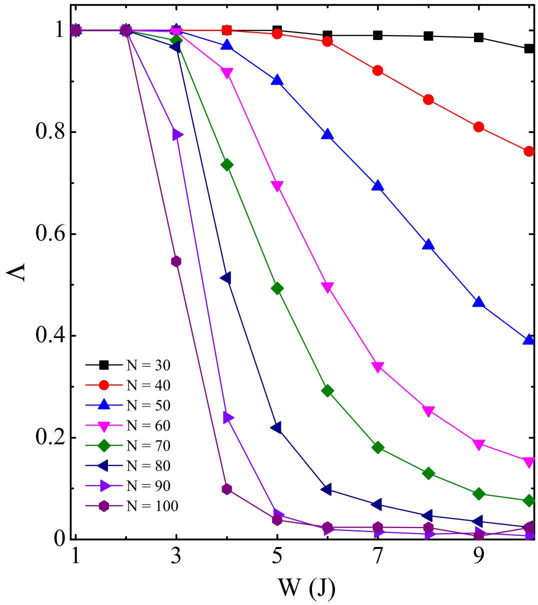


Fig. 3 End-to-end correlation amplitude Λ versus the on-site energy ε of spins 1 and $N+2$ (sender and receiver) averaged over 300 independent realizations of disorder for $N = 50$, $p = 0.5$, and defect energy $W/J = 1, 2, 4, 6$, and 8. Solid (dotted) curves correspond to correlated (uncorrelated) disorder. Data were obtained directly from Eqs. (5) and (6)

Fig. 4 End-to-end correlation amplitude Λ versus defect energy W for fixed $\varepsilon = W$, $p = 0.5$, and several channel sizes N averaged over 300 independent samples. Data were obtained directly from Eqs. (6) and (5). Note that as N increases past over $N \sim 50$, there shall be no useful QST protocol for $W > 2$



$N = 100$. Based on the Λ outcome and in the light of Eq. (8), for $N \sim 30$, QST should hold fairly well even for strong random (dimer-like) energies $W \sim 10J$. This fact, alone, is very appealing since early experiments in the field involves a smaller number of sites [59,60]. For longer chains, the end-to-end correlation amplitude remains intact when $W \leq 2J$, which is, still, a reasonable amount of disorder.

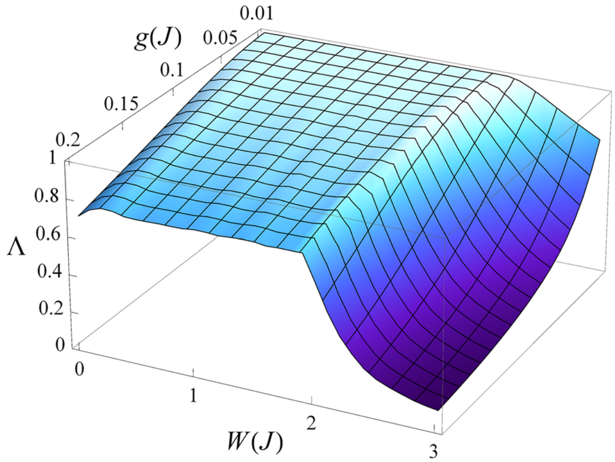


Fig. 5 End-to-end correlation amplitude Λ against defect energy W and g considering the full system, Hamiltonian (1), and correlated, dimerized disorder for $N = 30$, fixed $\omega_1 = \omega_{N+2} = \varepsilon = W$, and $p = 0.5$. We evaluated Λ by taking one of the eigenstates closest to energy level W (of the form of $|\psi^{\pm}\rangle$) and averaging the output over 300 independent samples for each g and W

So far we have been exploring the effective regime accounted by Hamiltonian (4), with the underlying assumption that g is perturbatively small. Noting in Eq. (8) that the transfer time scales with the inverse of the effective Rabi splitting Ω, we have $\tau \sim g^{-2}$ and therefore long times are generally required to perform that class of QST. Another constraint we should observe is that larger N and/or W tend to decrease the output of the sum in Eq. (6), thus decreasing Ω even further.

In Fig. 5, we evaluate the end-to-end correlation amplitude for one of the eigenstates of the full Hamiltonian, Eq. (1), closest to level W—the one featuring the highest overlap with spins 1 and N+2—and see how it bevahes upon slowly increasing g and W in a chain with $N = 30$ and correlated disorder. Again, we fix $\varepsilon = W$ for every W. Though Λ loses strength overall due to increasing mixing between the channel and the outer spins, there is an interesting behavior surrounding $W = 2J$, after which we observe a sudden decay of Λ for higher values of g. When $W < 2J$, the decrease in Λ occurs in a much slower pace. Curiously, from $W = 0$ to $2J$, Λ is roughly the same and thus we note that as long as W lies within the band of allowed energies of the noiseless system and provided $\varepsilon = W$, in terms of QST fidelity the chain qualitatively behaves like there is no disorder. This is irrespective of N as we have seen in Fig. 4 for the perturbative regime ($g \to 0$). We also would like to highlight in Fig. 5 that $\Lambda > 0.9$ for $W \leq 2J$ and *moderately* small values of g, say $0.1J$, what entails high-quality QST in more feasible timescales.

Last, we address the actual QST fidelity [Eq. (3)] which we evaluate over a fixed time interval and keep its maximum outcome for each disorder realization, further averaging them out. Here we follow the exact (non-perturbative) quantum dynamics of the channel attached to the sender and receiver sites. In Fig. 6, we display $F_{\max} = \max\{F(t)\}$ for $tJ \in [0, 2 \times 10^5]$ against tuning energy ε, averaged over 300 distinct realizations of correlated disorder, for $W/J = 1$ and 1.5, now setting an actual value

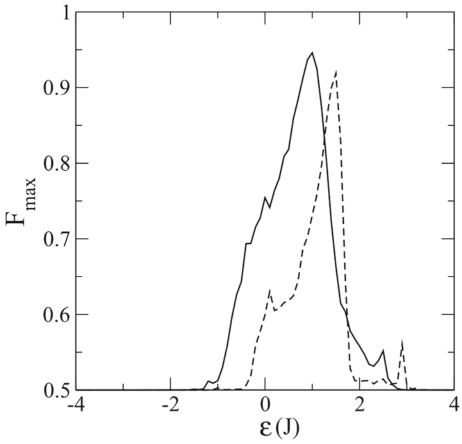


Fig. 6 Maximum fidelity $F_{\max}$ registered over time interval $tJ \in [0, 2 \times 10^5]$ versus the sender/receiver tuning energy ε for defect energies $W/J = 1$ (solid curve) and 1.5 (dotted curve) in the presence of correlated dimer-like defects with $p = 0.5$. Calculations were done for $g = 0.1J$ and $N = 50$ by exact numerical diagonalization of the full Hamiltonian, Eq. (2) and averaged over 300 independent samples

for $g/J = 0.1$. Though this is not a perturbatively small parameter, so as to justify Hamiltonian (4) up to great accuracy, there still remains the overall behavior spotted in Fig. 3 for W around the same order of magnitude. Furthermore, fidelities outcomes $F > 0.9$ can be assured whenever $\varepsilon \approx W$. Weak-coupling QST schemes [24,25] are thus shown to be robust against short-range correlated random defects provided the existence of "localization-free" zones in the spectrum of the channel.

4 Conclusions

In this work we have studied the role of short-range correlated (on-site) disorder—embodied by random dimer-like defects—in a QST protocol on a XY spin-1/2 chain featuring weak end bonds. We showed that a high-fidelity transmission can be attained provided the sender and receiver spins are both tuned to the disorder level $\varepsilon = W$ and that W is within the energy band of the defectless channel ($W \leq 2\,\mathrm{J}$), contrary to the uncorrelated single-defect case for which no QST should be supported at all when W is about the order of the channel couplings J.

Remarkably, we found that the protocol remains robust even for substantial values of $W \sim 10\,\mathrm{J}$ for moderate sizes $N \sim 30$ while still able to hold against the defects for much larger sizes provided $W \leq 2\,\mathrm{J}$. High fidelities are also achievable in the non-perturbative regime, with $g \sim 0.1\,\mathrm{J}$, for modest chain sizes. In a way, from $W = 0$ to $2\,\mathrm{J}$, the outer ends of the chain behaves as if there is no disorder within the channel once $\varepsilon = W$ at all times.

Our approach brings about further perspectives in the design of disorder-proof channels for quantum communication purposes. Next steps will be taken in optimizing QST and entanglement distribution protocols [19] over other classes of noise, and also considering different topologies [61,62].

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