Rogue waves in quantum lattices with correlated disorder

A. R. C. Buarque, W. S. Dias, G. M. A. Almeida, M. L. Lyra, and F. A. B. F. de Moura*
Instituto de Física, Universidade Federal de Alagoas, 57072-900 Maceió, Alagoas, Brazil

(Received 31 July 2022; accepted 19 January 2023; published 26 January 2023)

We investigate the outbreak of anomalous quantum wave-function amplitudes in a one-dimensional tight-binding lattice featuring correlated diagonal disorder. Such rogue-wave-like behavior is fostered by a competition between localization and mobility. The effective correlation length of the disorder is ultimately responsible for bringing the local disorder strength to a minimum, fueling the occurrence of extreme events of much higher amplitudes, especially when compared to the case of uncorrelated disorder. Our findings are valid for a class of discrete one-dimensional systems and reveal profound aspects of the role of randomness in rogue-wave generation.

DOI: 10.1103/PhysRevA.107.012425

I. INTRODUCTION

Rare and unpredictable events carrying huge impact are widespread in nature, from stock markets to physical sciences. Outliers may change the course of things more often than we are prone to think, sometimes leading to hazardous consequences. One example is the emergence of rogue waves in the ocean. The famous Draupner wave recorded in 1995 at a gas platform in Norway was twice as big as the significant wave height of the area. This happened to be the first scientific evidence of a rogue wave [1] and led to a burst of interest in the field as studies began to suggest that these extreme events would occur more frequently than assumed from ordinary Gaussian statistics [2]. About a decade later, Solli *et al.* introduced rogue waves in optics based on observations made on fiber supercontinuum generation in the presence of noise [3].

The analogy drawn between oceanic rogue waves and extreme instabilities in optics associated to long-tailed statistics set the stage for a number of theories aimed to explain the physical mechanisms behind their generation. This is usually done in the framework of the nonlinear Schrödinger equation describing the evolution of the wave envelope, with the modulation instability being one crucial nonlinear focusing mechanism responsible for rogue events. Much of the effort nowadays has been directed toward establishing whether and which linear or nonlinear processes play the biggest roles [4–11]. Oceanic rogue waves, for instance, may result from various mechanisms in action, such as constructive interference of random fields, modulational instability, and soliton modes, depending on sea and wind conditions [12,13]. Even though there is no definitive consensus on that matter, neither robust ways to predict where and when rogue waves will occur, noise and randomness seem to be key ingredients for their occurrence [11,14,15] (deterministic rogue waves are also discussed in Refs. [4,5]).

Some degree of disorder is paramount for generating rogue waves via linear mechanisms in particular [11,14–25]. Very recently, this was addressed in the context of quantum walks [25]. Therein, the authors primarily sought to explore the (hitherto elusive) relationship between Anderson localization and rogue wave manifestation. They reported a minimal disorder threshold $\propto N^{-1/2}$ (N being the number of sites) above which rogue waves were created, whereas intermediate disorder levels would maximize the chances of seeing one due to a proper balance between trapping and mobility (see also Ref. [21]). Such results, besides bringing the rich subject of rogue waves up to the realm of quantum transport, add important elements to the issue of the actual role played by randomness.

With those ideas in mind, here we set out to track the dynamics of rogue waves on a single quantum particle propagating in a lattice featuring correlated disorder. Anderson localization theory settles that all single-particle eigenstates are exponentially localized for any amount of uncorrelated disorder in one and two dimensions [26]. This can be violated, however, when the disorder displays intrinsic correlations [27–29]. Scale-free correlations, for instance, are known to support a metal-insulator transition with sharp mobility edges [27].

In this article our goal is to investigate the development of sudden, anomalous quantum amplitudes due to the interplay between localized and extended states. Indeed, long-range correlated fluctuations in the random input phases was recently shown to produce rogue waves way above the threshold in an experiment on linear light diffraction in 1D [22].

We look at a particular kind of correlated disorder in which a single parameter is able to control the typical correlation length and, in turn, the local disorder strength. The latter is found to be a crucial factor underlying the generation of the rogue waves for it sets up the right amount of wave-function mobility. This boosts not only the number of occurrences, but also the average rogue wave amplitude.

*fidelis@fis.ufal.br

II. MODEL

We consider a single quantum particle propagating in a one-dimensional array described by the tight-binding Hamiltonian

$$H = J\sum_{n=1}^{N}(|n\rangle\langle n+1| + |n+1\rangle\langle n|) + \sum_{n}^{N}\varepsilon_n|n\rangle\langle n|, \quad (1)$$

where J is the nearest-neighbor hopping strength, ε_n is the on-site potential, and states $|n\rangle$ represent the location of the particle and span the whole Hilbert space. As such, an arbitrary quantum state can be written as $|\Psi\rangle = \sum_n \psi_n|n\rangle$, with the normalization condition $\sum_n|\psi_n|^2 = 1$. The time evolution of the wave function ψ_n is given by the Schrödinger equation

$$i\frac{d}{dt}\psi_n = \psi_{n+1} + \psi_{n-1} + \varepsilon_n\psi_n, \quad (2)$$

where $\hbar = J = 1$ without loss of generality.

Here we go beyond the standard case of uncorrelated disorder and consider that the local potentials ε_n are embedded with correlations, as given by

$$\varepsilon_n = \sum_m \frac{Z_m}{(1 + d_{n,m}/A)^2}, \quad (3)$$

where Z_m is a random number $\in [-1, 1]$, $d_{n,m}$ is the Euclidean distance between sites n and m, and A controls the correlation length of the series. As we impose periodic boundary conditions $|N+1\rangle = |1\rangle$, in order to properly evaluate $d_{n,m}$ it is convenient to set the location of each site n through the coordinates $(x_n = R\sin\theta_n, y_n = R\cos\theta_n)$, with $\theta_n = (2\pi/N)n$, rendering $d_{n,m} = \sqrt{(x_n - x_m)^2 + (y_n - y_m)^2}$. The disordered sequence is further normalized to have zero mean and unit variance.

To see how the above disorder configuration plays out with the correlation parameter A, in Fig. 1(a) we plot the autocorrelation function $C(r) = \mathrm{cov}(\varepsilon_i, \varepsilon_{i+r}) = \sum_{i=1}^{N-r}\varepsilon_i\varepsilon_{i+r}/(N-r)$. Upon increasing A, $C(r)$ exhibits a slower decay, as expected from Eq. (3). We are then able to set an effective correlation length L_c by fitting $C(r) \propto e^{-r/L_c}$. Figure 1(b) shows that $L_c \propto A$, thereby affirming what the latter stands for.

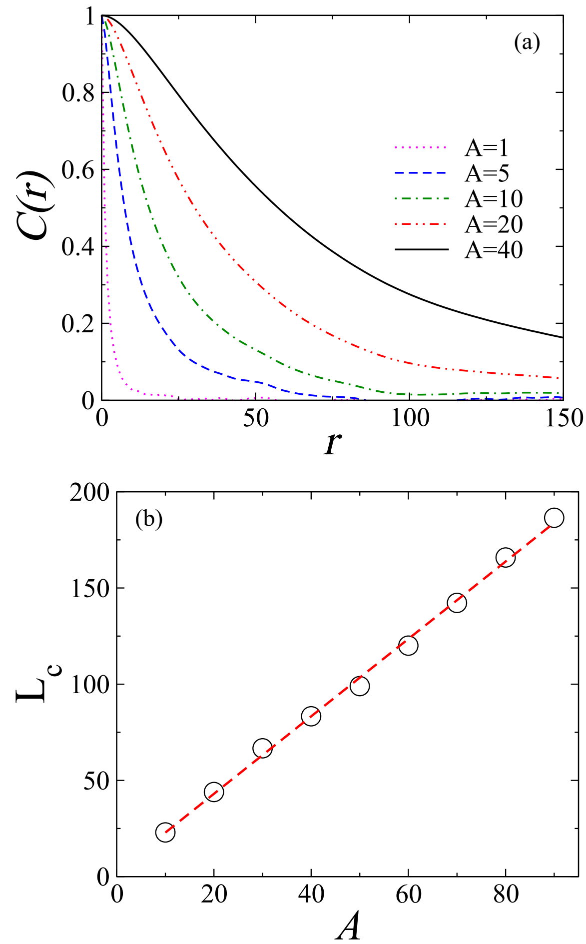

FIG. 1. (a) Autocorrelation function $C(r) = \mathrm{cov}(\varepsilon_i, \varepsilon_{i+r})$ vs r for many values of the correlation parameter A, averaged over 100 independent samples. (b) Effective correlation length $L_c \propto A$.

III. RESULTS

We are ready to search for the occurrence of rogue waves and address the role of the correlated disorder. In all simulations below, the set of equations given in Eq. (2) is numerically solved by employing a high-order Taylor expansion of the time evolution operator:

$$U(\Delta t) = \exp(iH\Delta t) = 1 + \sum_{l=1}^{n_0}\frac{(iH\Delta t)^l}{l!}, \quad (4)$$

with time step $\Delta t = 0.01\ \mathrm{J}^{-1}$ and $n_0 = 20$, which is enough to produce smooth outcomes and keep the norm conserved during the whole time interval. In order to avoid ambiguity between a rogue wave event and trapping of the wave function due to Anderson localization, we set $\psi_n(t=0) = 1/\sqrt{N}$ for all n [25].

Figure 2 shows typical scenarios of rogue waves, as told by the probability amplitude of the particle wave function $|\psi_n(t)|^2$, for different values of the correlation strength A. For weak correlations the evolution is characterized by sparse, low-amplitude waves that eventually add up to produce the rogue events, as shown in Fig. 2(a), around $t = 3000/\mathrm{J}$. As the correlation strength is increased, the background becomes more inhomogeneous—a fundamental trait in the generation of rogue waves via linear processes [11]. There sets in distinct amplitude domains in space, indicating that A is pushing for weaker local fluctuations in the disorder distribution (to be addressed in a moment). As a consequence, rogue waves of exceptionally higher amplitudes are likely to occur [cf. Fig. 2(d)].

We will not be using here any particular rogue-wave criteria. One measure employed in various contexts is the significant wave height, commonly defined as twice the average of largest one-third of values in a data set [12]. An event is thus considered a rogue wave whenever it beats that level. Considering our initial state and the results obtained in Ref. [25], that deals with a similar class of problem, such a threshold would be of the order of $1/N$. As our following analysis is built on extreme-value statistics, the data is heavily loaded with amplitudes well above that mark.

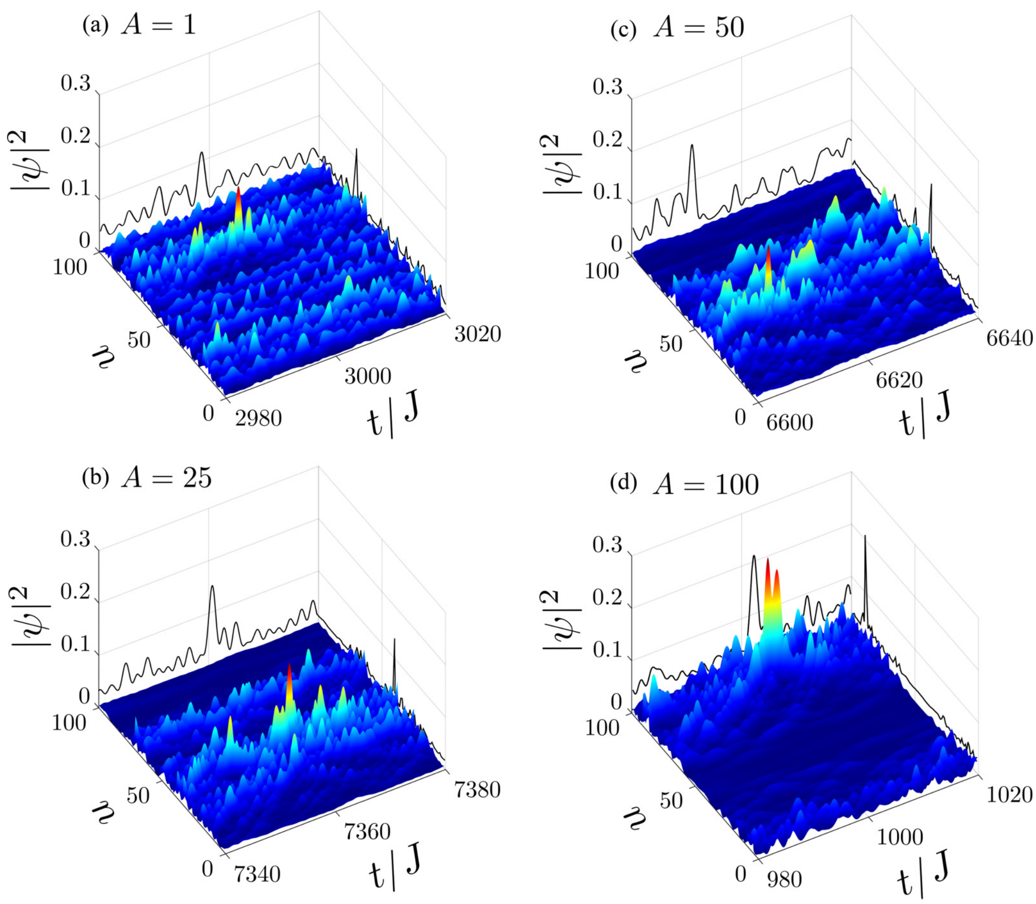

FIG. 2. Snapshots of the space-time evolution of the particle wave-function probability highlighting typical rogue wave events for different correlation strengths: (a) $A = 1$, (b) $A = 25$, (c) $A = 50$, and (d) $A = 100$. Two-dimensional graphs display spatial and temporal cross sections for the largest amplitudes. Time is expressed in units of J^{-1}.

Let us now obtain one of the limiting extreme-value distributions, according to the Fisher-Tippett-Gnedenko theorem [30]. In order to do so, we pick the maximum wave-function amplitude in space at each time step for several independent realizations of the disorder. The stacked outcomes are seen in the probability density functions (PDFs) shown in Fig. 3 for different degrees of correlation A. Note that this effective correlation length indeed brings forth higher amplitudes. However, there is a threshold value for A above which the right tail of the distribution begins to deflate (rare, extreme rogue waves can still develop). To learn more about this nonmonotonic relationship between the rogue-wave maximum amplitudes and the correlation parameter A, we portray in Fig. 4 the maximum amplitudes $|\psi|^2_{\max}$ allowed at a probability level just above 10^{-7} (see Fig. 3). This is done in order to avoid extremely rare outcomes. The scaling with N in Fig. 4 is employed so we can filter out finite-size effects and focus on the role of the correlation degree only. Compared with what one would obtain from uncorrelated series of the potential ε_n (cf. dashed curve in Fig. 4), the rogue waves can reach almost twice as large amplitudes when supported by the correlated disorder.

We also find that all the skewed PDFs shown in Fig. 3 belong to the Gumbel class of extreme-value distributions, fitted by $P(x) \propto \exp[-(\alpha x + \beta \exp(-\alpha x))]$. Similar behavior is found in quantum walks featuring uncorrelated disorder [25], meaning that the wave-function fluctuations are well described by processes involving independent and identically distributed random variables. The Gumbel distribution applies whenever the parent distribution is of the form $p(x) \sim e^{-x^{\delta}}$, with $\delta > 0$ [30]. So that is the expected output from a sum of random complex amplitudes (phasors) in the limit of a large number of contributions, where the Rayleigh exponential law for $|\psi|^2$ is obtained ($\delta = 1$) [31]. In Fig. 5 we show the parent distributions $p(|\psi|^2)$ for distinct values of A obtained numerically. For $A > 1$ the distributions are essentially exponential (rendering a straight line in the log-linear plot), aside from the pronounced tail signaling the occurrence of rogue waves. The exception is noted when $A = 1$, whose distribution displays a bent tail. This suggests that something in the random phasor sum underlying the quantum state evolution is preventing it to reach the Rayleigh regime. Note that a delocalized input leads to

$$\psi_n(t) = \sum_{n'} \left(\sum_k e^{-iE_k t} v_{k,n} v_{k,n'} \right), \quad (5)$$

where $v_{k,n}$ is the kth eigenstate of the Hamiltonian projected onto n and E_k its corresponding eigenvalue. The expression above can be seen as a sum of random phasor sums [31]. Now,

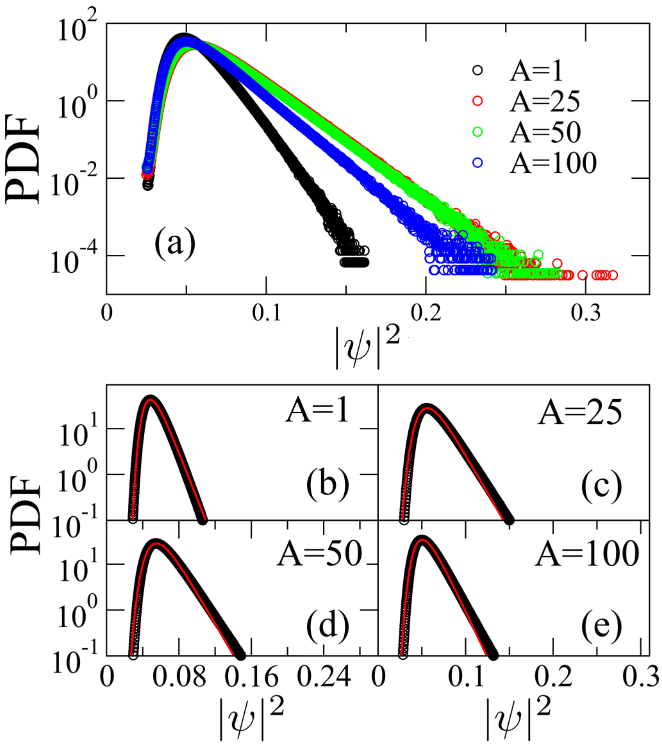

FIG. 3. Extreme-value distributions generated by sorting up the maximum values of $|\psi_n|^2$ at each time step during the whole evolution (up to $t = 15\,000/\mathrm{J}$ with $\Delta t = 0.01/\mathrm{J}$) for 500 independent samples of a disordered chain with $N = 100$ sites when $A = 1, 25, 50, 100$. Bottom panels show the fitted Gumbel distribution (red curve) of the form $P(x) \propto \exp[-(\alpha x + \beta \exp(-\alpha x))]$.

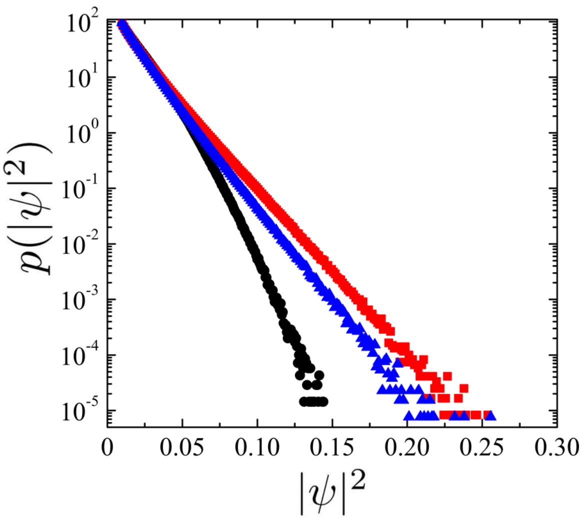

FIG. 5. Distribution of $|\psi|^2$ accumulated from an ensemble of 10^3 independent realizations of disorder for a chain with $N = 100$ sites, each evolving up to $t = 15\,000/\mathrm{J}$. Correlation parameters are $A = 1$ (circles), $A = 30$ (triangles), and $A = 90$ (diamonds), respectively.

given $A = 1$, the system is effectively under the influence of strong uncorrelated disorder. As such, exponentially localized eigenstates will lead those phasor sums carrying $n \approx n'$ to other distributions other than Rayleigh. Further details will be reported elsewhere [32].

We point out that a finite correlation length should maintain the asymptotic Gumbel form whenever the correlation length within the series remains much smaller than its length [30]. And we must distinguish two correlation lengths that appear in our model. The first one is over the random potential that proportionally grows the correlation parameter A. The second (and most important) one is the localization length of the Hamiltonian eigenstates, which depends on the local disorder variance, as we will see shortly.

The regularity of rogue wave events occurring on systems featuring static disorder can be predicted to some extent based on the energy resonance conditions across the lattice. Either very weak or very strong levels of disorder should suppress the onset of anomalous wave-function fluctuations. Some studies report that an optimal balance between localization

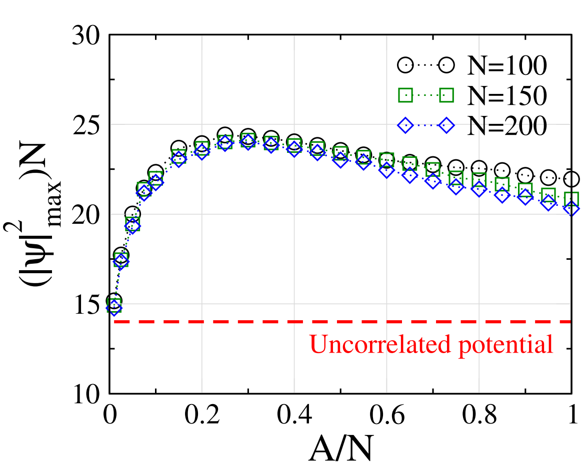

FIG. 4. Maximum wave-function probability amplitude $|\psi|^2_{\max}$ scaled with N (excluding exceptionally rare events) for a range of A/N values.

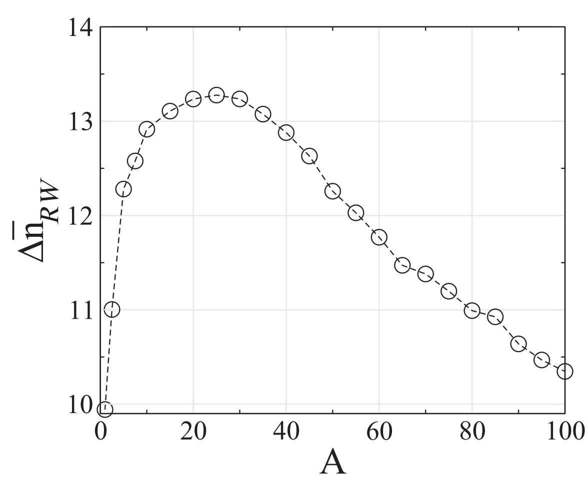

FIG. 6. Mean number of sites actively involved in the generation of rogue waves $\Delta\bar{n}_{RW}$ vs correlation strength A, averaged over 500 independent realizations of disorder, for times up to $t = 15\,000/\mathrm{J}$ and $N = 100$. Note that the values of A that give the highest $\Delta\bar{n}_{RW}$ also lead to outbreaks of very intense rogue waves.

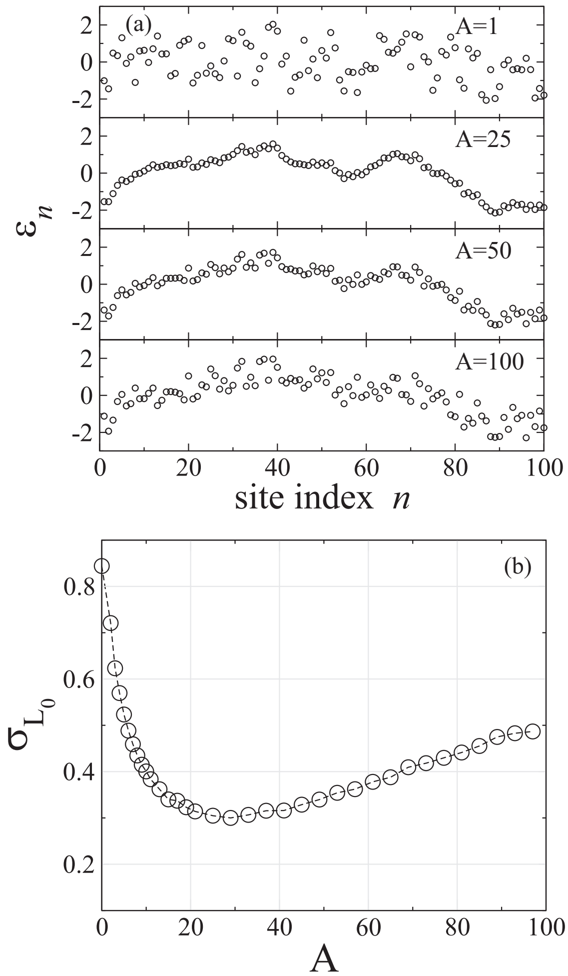

FIG. 7. (a) Series of the on-site potential ε_n considering $A = 1, 25, 50$, and 100. (b) Local standard deviation σ_{L_0} as a measure of the local disorder strength. Weaker local fluctuations are obtained at intermediate values of A, the same range that renders better chances of observing rogue waves of higher amplitudes.

and mobility can maximize their chances to happen [21,25]. Here, this balance is effectively imposed by the correlation level A. To show there is indeed a coordinated dynamics taking place, let us track how many sites, for a given disordered sample, are mostly involved in those extreme events. Defining $\pi(n)$ as the probability of a rogue occurring at site n, we compute the mean number of participating sites as $\Delta\bar{n}_{RW} = 1/[\sum_n \pi(n)^2]$, which ranges from 1 (fully biased) to N (equally distributed). Figure 6 displays this quantity against the correlation parameter A. Again, the nonmonotonic behavior is clear and indicates that intermediate values of A implies more sites actively participating in the generation of rogue waves. In turn, waves of higher amplitudes become more likely (cf. Figs. 3 and 4).

Last, we complement the above discussion by taking a look back at the potential series given by Eq. (3). Figure 7(a) shows how it typically sets up along the lattice. For $A = 1$, the series features a white noise (almost uncorrelated) profile. Then, at $A = 25$ local fluctuations are drastically reduced. As we further increase the effective correlation length, a rougher landscape is obtained, but still carrying a predominant harmonic component. To put all that together, let us compute the local disorder strength in terms of the local standard deviation $\sigma_{L_0} = (\sum_{k=1}^{M} \sigma_{k,L_0})/M$ within a segment with L_0 sites, where

$$\sigma_{k,L_0} = \sqrt{\sum_{n=(k-1)L_0+1}^{kL_0} \frac{\varepsilon_n^2}{L_0} - \left(\sum_{n=(k-1)L_0+1}^{kL_0} \frac{\varepsilon_n}{L_0}\right)^2}. \quad (6)$$

Results are shown in Fig. 7(b), where we are able to confirm once for all that intermediate values of A makes for minimum local fluctuations, which is consistent with the smooth potential landscape seen in Fig. 7(a). This increases the localization length of the eigenstates of the Hamiltonian (that typically scales as $1/\sigma_{L_0}^2$) while keeping it much smaller than the size of the system.

We emphasize it is the local variance (not the global one) that stands as a proper measure of the disorder strength. This is supported by the fact that the statistical properties of the extreme events are closely correlated to the nonmonotonic dependence of the local disorder variance on the correlation parameter A even when the global disorder is fixed. While these two disorder measures coincide for uncorrelated potentials, they capture distinct aspects of the randomness in the presence of correlations. Therefore the reported results here cannot be derived from a trivial rescaling of disorder parameters.

IV. CONCLUDING REMARKS

We explored the intrinsic role played by correlated disorder on the emergence of rogue waves in a simple quantum tight-binding model. The amplitude of the particle wave function was found to exhibit strong anomalous fluctuations inherently unpredictable in time and space. An approach based on extreme-value statistics revealed that such fluctuations follow a Gumbel distribution, belonging to the same class as those reported in a quantum walk featuring uncorrelated disorder [25].

We learned that intermediate values of the correlation parameter A, acting here as an effective correlation length, brings down the local disorder strength. This properly enhances the mobility of the wave function (by spanning eigenstates with larger localization lengths) and so the number of sites on which rogue-wave events take place, which in turn amplifies their characteristic amplitude, almost twice as large when compared to uncorrelated scenarios [21,22,25].

While disorder *must* be present to promote random fluctuations of the wave function, the underlying single-particle eigenstates must be wide enough in order to allow the linear superposition of the many components needed to produce localized waves in space and time. A general result we can establish is that rogue waves are expected to occur more often in the regime of weak disorder.

Our findings are general to a class of 1D discrete disordered systems and bring about another perspective on their dynamics as well as reveal fundamental aspects of the role of randomness in the generation of rogue waves. They are supported by results on random branched Hamiltonian flows, where weak correlated potentials are known to have a major influence in the burst of extreme fluctuations associated to caustics [16,33–35]. Indeed, their universal statistics are applied to a wide range of random potentials [16,34]. A key point we unveil here is that the localization length is the relevant scale dictating the occurrence of the rogue waves, regardless of the profile of the correlated series. In fact, when the correlation is nearly absent the distribution of $|\psi|^2$ does not approach the Rayleigh exponential form [see Fig. 5]. In this case, the local disorder variance is high enough to render a state evolution dominated by a few strongly localized modes. If we make an analogy with established results in statistical optics [31], too much disorder inhibits the usual fully developed speckle regime. This requires the contribution of a large number of random complex amplitudes as well as statistical independence between them [31]. The specific form of the distributions that can be obtained on a strongly disordered lattice deserves further investigation [32].

Deviations from the Rayleigh law are also expected if decoherence is involved. In [35] Metzger *et al.* report a transition from the exponential distribution to a log-normal distribution when phase coherence of the waves is lost. In a general quantum setting this could investigated in terms of quantum dynamical maps. Future works should also address the role of particle interaction in extreme events, especially in the many-body regime where caustics yields to heavy tailed statistics [36]. In this context, rogue wave events can be associated to localization in Fock space. Despite the dynamics being much more complex, information theory measures [37] may be in place to access the onset of such intrinsic turbulent behavior.

ACKNOWLEDGMENT

This work was supported by CAPES, CNPq, and FAPEAL (Alagoas State Agency).

[1] S. Harvey, in *Rogue Waves 2004*, edited by M. Olagnon and M. Prevosto (Ifremer Publications, Brest, France, 2004).
[2] J. M. Dudley, G. Genty, A. Mussot, A. Chabchoub, and F. Dias, Rogue waves and analogies in optics and oceanography, Nat. Rev. Phys. **1**, 675 (2019).
[3] D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, Optical rogue waves, Nature (London) **450**, 1054 (2007).
[4] C. Bonatto, M. Feyereisen, S. Barland, M. Giudici, C. Masoller, J. R. R. Leite, and J. R. Tredicce, Deterministic Optical Rogue Waves, Phys. Rev. Lett. **107**, 053901 (2011).
[5] F. Baronio, M. Conforti, A. Degasperis, S. Lombardo, M. Onorato, and S. Wabnitz, Vector Rogue Waves and Baseband Modulation Instability in the Defocusing Regime, Phys. Rev. Lett. **113**, 034101 (2014).
[6] G. Xu, A. Chabchoub, D. E. Pelinovsky, and B. Kibler, Observation of modulation instability and rogue breathers on stationary periodic waves, Phys. Rev. Res. **2**, 033528 (2020).
[7] N. Akhmediev, J. M. Soto-Crespo, and A. Ankiewicz, How to excite a rogue wave, Phys. Rev. A **80**, 043818 (2009).
[8] M. Onorato, A. R. Osborne, M. Serio, and S. Bertone, Freak Waves in Random Oceanic Sea States, Phys. Rev. Lett. **86**, 5831 (2001).
[9] L. H. Ying, Z. Zhuang, E. J. Heller, and L. Kaplan, Linear and nonlinear rogue wave statistics in the presence of random currents, Nonlinearity **24**, R67 (2011).
[10] A. Safari, R. Fickler, M. J. Padgett, and R. W. Boyd, Generation of Caustics and Rogue Waves from Nonlinear Instability, Phys. Rev. Lett. **119**, 203901 (2017).
[11] F. T. Arecchi, U. Bortolozzo, A. Montina, and S. Residori, Granularity and Inhomogeneity Are the Joint Generators of Optical Rogue Waves, Phys. Rev. Lett. **106**, 153901 (2011).
[12] C. Kharif, E. Pelinovsky, and A. Slunyaev, *Rogue Waves in the Oceans* (Springer-Verlag, Berlin, Heidelberg, 2009).
[13] K. Dysthe, H. E. Krogstad, and P. Müller, Oceanic rogue waves, Annu. Rev. Fluid Mech. **40**, 287 (2008).
[14] R. Höhmann, U. Kuhl, H.-J. Stöckmann, L. Kaplan, and E. J. Heller, Freak Waves in the Linear Regime: A Microwave Study, Phys. Rev. Lett. **104**, 093901 (2010).
[15] C. Liu, R. E. C. van der Wel, N. Rotenberg, L. Kuipers, T. F. Krauss, A. Di Falco, and A. Fratalocchi, Triggering extreme events at the nanoscale in photonic seas, Nat. Phys. **11**, 358 (2015).
[16] J. J. Metzger, R. Fleischmann, and T. Geisel, Statistics of Extreme Waves in Random Media, Phys. Rev. Lett. **112**, 203903 (2014).
[17] A. Mathis, L. Froehly, S. Toenger, F. Dias, G. Genty, and J. M. Dudley, Caustics and rogue waves in an optical sea, Sci. Rep. **5**, 12822 (2015).
[18] M. Mattheakis, I. Pitsios, G. Tsironis, and S. Tzortzakis, Extreme events in complex linear and nonlinear photonic media, Chaos, Solitons, and Fractals **84**, 73 (2016).
[19] S. Derevyanko, Anderson localization of a one-dimensional quantum walker, Sci. Rep. **8**, 1795 (2018).
[20] M. Peysokhan, J. Keeney, and A. Mafi, Impact of spatial correlation in fluctuations of the refractive index on rogue wave generation probability, Opt. Lett. **44**, 2244 (2019).
[21] D. Rivas, A. Szameit, and R. A. Vicencio, Rogue waves in disordered 1D photonic lattices, Sci. Rep. **10**, 13064 (2020).
[22] C. Bonatto, S. D. Prado, F. L. Metz, J. R. Schoffen, R. R. B. Correia, and J. M. Hickmann, Super rogue wave generation in the linear regime, Phys. Rev. E **102**, 052219 (2020).
[23] H. Frostig, I. Vidal, R. Fischer, H. H. Sheinfux, and Y. Silberberg, Observation of rogue events in non-Markovian light, Optica **7**, 864 (2020).
[24] R. da Silva and S. D. Prado, A simple study of the correlation effects in the superposition of waves of electric fields: The emergence of extreme events, Phys. Lett. A **384**, 126231 (2020).
[25] A. R. C. Buarque, W. S. Dias, F. A. B. F. de Moura, M. L. Lyra, and G. M. A. Almeida, Rogue waves in discrete-time quantum walks, Phys. Rev. A **106**, 012414 (2022).
[26] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions, Phys. Rev. Lett. **42**, 673 (1979).
[27] F. A. B. F. de Moura and M. L. Lyra, Delocalization in the 1D Anderson Model with Long-Range Correlated Disorder, Phys. Rev. Lett. **81**, 3735 (1998).
[28] O. Dietz, U. Kuhl, H.-J. Stöckmann, N. M. Makarov, and F. M. Izrailev, Microwave realization of quasi-one-dimensional

systems with correlated disorder, Phys. Rev. B **83**, 134203 (2011).

[29] F. Izrailev, A. Krokhin, and N. Makarov, Anomalous localization in low-dimensional systems with correlated disorder, Phys. Rep. **512**, 125 (2012).

[30] S. N. Majumdar, A. Pal, and G. Schehr, Extreme value statistics of correlated random variables: A pedagogical review, Phys. Rep. **840**, 1 (2020).

[31] J. Goodman, *Speckle Phenomena in Optics: Theory and Applications*, Press Monographs (SPIE Press, Bellingham, WA, 2020).

[32] G. M. A. Almeida, A. M. C. Souza, M. L. Lyra, and F. A. B. F. de Moura, Rogue-wave statistics on Anderson chains (unpublished).

[33] L. Kaplan, Statistics of Branched Flow in a Weak Correlated Random Potential, Phys. Rev. Lett. **89**, 184103 (2002).

[34] J. J. Metzger, R. Fleischmann, and T. Geisel, Universal Statistics of Branched Flows, Phys. Rev. Lett. **105**, 020601 (2010).

[35] J. J. Metzger, R. Fleischmann, and T. Geisel, Intensity Fluctuations of Waves in Random Media: What Is the Semiclassical Limit? Phys. Rev. Lett. **111**, 013901 (2013).

[36] W. Kirkby, Y. Yee, K. Shi, and D. H. J. O'Dell, Caustics in quantum many-body dynamics, Phys. Rev. Res. **4**, 013105 (2022).

[37] M. Shavit and G. Falkovich, Singular Measures and Information Capacity of Turbulent Cascades, Phys. Rev. Lett. **125**, 104501 (2020).