Finite-size scaling and disorder effect on the transmissivity of multilayered structures with metamaterials

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Abstract: We investigate the influence of metamaterials on the scaling laws of the transmission on multilayered structures composed of random sequences of ordinary dielectric and metamaterial layers. The spectrally averaged transmission in a frequency range around the fully transparent resonant mode is shown to decay with the total number of layers as $1/N$. Such thickness dependence is faster than the $1/N^{1/2}$ decay recently reported to take place in random sequences of ordinary dielectric slabs. The interplay of strong localization and the emergence of resonant modes within the gap leads to a non-monotonous disorder dependence of the transmission that reaches a minimum at an intermediate disorder strength.

OCIS codes: (230.4170) Multilayers; (160.3918) Metamaterials; (120.7000) Transmission

References and links

1. V. G. Veselago, "The electrodynamics of substances with simultaneously negative values of permissivity and permeability," Sov. Phys. Usp. **10**, 509-514 (1968).
2. J. B. Pendry, A. J. Holden, W. J. Stewart and I. Youngs, "Extremely low frequency plasmons in metallic mesostructures," Phys. Rev. Lett. **76**, 4773 (1996).
3. P. Kolinko and D. R. Smith, "Numerical study of electromagnetic waves interacting with negative index materials," Opt. Express **11**, 640 (2003).
4. K. Guven, M. D. Caliskan and E. Ozbay, "Experimental observation of left-handed transmission in a bilayer metamaterial under normal-to-plane propagation," Opt. Express **30**, 8685 (2006).
5. Z. Tang, R. Peng, D. Fan, S. Wen, H. Zhang and L. Qian, "Absolute left-handed behaviors in a triangular elliptical-rod photonic crystal," Opt. Express **13**, 9796 (2005).
6. D. R. Smith and N. Kroll, "Negative refractive index in left-handed materials," Phys. Rev. Lett. **84**, 4184 (2000).
7. P. Markǒs and C. M. Soukoulis, "Transmission properties and effective electromagnetic parameters of double negative metamaterials," Opt. Express **11**, 649 (2003).
8. J. B. Pendry, "Negative refraction makes a perfect lens," Phys. Rev. Lett. **85**, 3966 (2000)
9. J. A. Kong, B.-I. Wu and Y. Zhang, "A unique lateral displacement of a gaussian beam transmitted through a slab with a negative permittiviy and permeability," Microwave Opt. Technol. Lett. **33**, 136-139 (2002).
10. E. Yablonovitch, "Inhibited spontaneous emission in solid-state physics and electronics," Phys. Rev. Lett. **58**, 2059 (1987)
11. Liang Wu, Sailing He and Linfang Shen, "Band structure for a one-dimensional photonic crystal containing left-handed materials," Phys. Rev. B **67**, 235103 (2003)
12. Jensen Li, Lei Zhou, C. T. Chan and P. Sheng, "Photonic band gap from a stack of positive and negative index materials," Phys. Rev. Lett. **90**, 083901 (2003)
13. J. A. Monsoriu, R. A. Depine, M. L. Martínez-Ricci and E. Silvestre, "Interaction between non-Bragg band gaps in 1D metamaterials photonic crystals," Opt. Express **14**, 12958 (2006).
14. Y. Yuan, L. Ran, J. Huangfu, H. Chen, L. Shen and J. Au Kong, "Experimental verification of zero order bandgap in a layered stack of left-handed and right-handed materials," Opt. Express **14**, 2220 (2006)
15. Mher Ghulinyan, "Formation of optimal-order necklace modes in one-dimensional random photonic superlattices," Phys. Rev. A **76**, 013822 (2007)

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16. Mher Ghulinyan, ”Pediodic oscillations in transmission decay of Anderson localized one-dimensional dieletric system,” Phys. Rev. Lett. **99**, 063905 (2007)
17. J. Bertolotti, S. Gottardo, D. S. Wiersma, M. Ghulinyan and L. Pavesi, ”Optical necklace states in Anderson localized 1D systems,” Phys. Rev. Lett. **94**, 113903 (2005)
18. E. M. Nascimento, F. A. B. F. de Moura and M. L. Lyra, ”Scaling laws for the transmission of random binary dielectric multilayered structures,” Phys. Rev. B **76**, 115120 (2007)
19. J. Bertolotti, M. Galli, R. Sapienza, M. Ghulinyan, S. Gottardo, L. C. Andreani, L. Pavesi and D. S. Wiersma, ”Wave transport in random systems: Multiple resonance character of necklace modes and their statistical behavior,” Phys. Rev. E **74** 035602(R) (2006)
20. M. Titov and H. Schomerus, ”Nonuniversality of Anderson Localization in Short-Range Correlated Disorder,” Phys. Rev. Lett. **95** 126602 (2005)
21. A. A. Asatryan, L. C. Botten, M. A. Byrne, V. D. Freilikher, S. A. Gredeskul, I. V. Shadrivov, R. C. McPhedran and Y. S. Kivsharet, ”Suppression of Anderson localization in disordered metamaterials,” Phys. Rev. Lett. **99** 193902 (2007)

1. Introduction

Metamaterials are optical structures designed to present both negative permissivity and negative permeability. This characteristic leads to unusual electromagnetic phenomena, for instance, $\mathbf{E} \times \mathbf{H}$ lies along the direction of $-\mathbf{k}$ for propagating plane waves. A theoretical study of metamaterials was firstly suggested a half-century ago by Veselago [1]. However, the more recent proposal of Pendry [2] for the development of such materials stimulated a wealthy of theoretical and experimental researches on this subject [3, 4, 5, 6, 7].

Novel and fascinating phenomena have been shown to emerge in structures that include metamaterials. Pendry comproved that a slab of negative refractive index material has the power to focus all Fourier components of a $2D$ image. He named this device “superlens”[8]. This structure also provides a negative lateral displacement of an obliquely incident Gaussian beam[9].

Multilayer structures have been attracting a widespread interest because of their deep implications on both fundamental and technological aspects. A periodically modulated dielectric function is the simplest way to construct a one-dimensional photonic crystal (PC) [10]. For a PC containing metamaterials, some unusual phenomena such as spurious modes with complex frequencies, discrete modes and photon tunneling modes are observed in the band structure [11]. The mixing of metamaterial and ordinary dielectric slabs creates new features such as the rising of a non-Bragg band-gap called *zero-$\bar{n}$ gap* [12, 13], which emerges naturally when the volume average of the effective refractive index equals to zero. Such gap has a special characteristic: it remains unchanged when the lattice constant varies. Experimental verifications of *zero-$\bar{n}$ gap* were made using *double-S shaped* metamaterials [14].

In $1D$ disordered multilayered structures, multiple incoherent superposition of waves reflected by the layers interfaces give rises to the exponential localization of most of the electromagnetic modes within the structure. A few resonant necklace modes may remain extended[15, 16, 19]. This phenomenon is similar to the electronic eigenfunction localization in disordered atomic chains and is usually termed as the Anderson localization of light. In a recent paper [18], we investigated a 1D multilayered structure composed of a random sequence of N ordinary dielectric slabs with refraction indices satisfying the Bragg condition. An experimental observation of resonant transport in similar structures was reported by Bertolotti *et al* [19]. We showed that the half-wavelength mode is insensitive to disorder and fully transparent. The spectrally averaged transmission in a frequency range around this mode decays as $1/N^{1/2}$ and the localization length diverges quadratically as this resonant mode is approached. In the vicinity of the quarter-wavelength mode, the localization length diverges logarithmically and the frequency averaged transmission exhibits an stretched exponential dependence on the total thickness. These scaling laws were discussed in the context of the Anderson localization of electrons in chains with off-diagonal and correlated disorder. It was recently demonstrated theo-

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retically that correlations may indeed strongly modify the transmission properties of electronic wires and waveguides in small energy windows [20].

New features have been also reported when metamaterials are introduced in disordered optical system. Asatryan *et. al.* [21] studied wave propagation in mixed $1D$ disordered stacks of alternating right- and left-handed layers and revealed that the introduction of metamaterials substantially suppresses the Anderson localization. At long wavelengths, the localization length in mixed stacks is several orders of magnitude larger than in normal structures.

In this work, we study the influence of metamaterials on the scaling laws for the transmission of an electromagnetic plane wave inciding normally on a multilayer random structure. We will show that, for layers do not satisfying the Bragg condition, the transmission peak around the fully transmitting mode is quite sensitive to disorder. The spectrally averaged transmission develops a faster decay as a function of the total number of layers when compared with random structures composed just of ordinary dielectric slabs. The scaling behavior of the localization length remains unchanged. The interplay between the faster finite-size scaling and the emergence of resonance necklace modes within the gap leads to a non-monotonous dependence of the transmission as a function of the disorder strength.

2. Model and formalism

For a monochromatic plane wave of angular frequency ω propagating along the z axis direction, the electric and magnetic fields at the interfaces of a dielectric slab of thickness d can be numerically obtained by a transfer matrix formalism. Assuming a normally incident wave, linearly polarized in such a way that the electric field amplitude can be written as $\vec{E}(z) = E(z)\hat{x}$, the relation between the electric and magnetic field at the interface placed at $z = z_1$ and the fields at the interface placed at $z = z_1 + d$ can be expressed in transfer matrix form as

$$\begin{pmatrix} E_1 \\ B_1 \end{pmatrix} = M_2 \begin{pmatrix} E_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} \cos\delta & \frac{i}{p}\sin\delta \\ ip\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} E_2 \\ B_2 \end{pmatrix} \tag{1}$$

where the phase change $\delta = \pm|n|\omega d/c$ [the choice of sign corresponding to right-handed (RH) and left-handed (LH) slabs, respectively], $|n|$ is the absolute refractive index of the medium, and $p = \sqrt{\frac{\varepsilon}{\mu}}$, where ε and μ are the dielectric constant and magnetic permeability, respectively. For a stratified medium consisting of a sequence of N dielectric layers, the boundary conditions across the interface require the continuity of the parallel components of the fields. This relation can be written as a product of individual transfer matrices as

$$\begin{pmatrix} E_0 \\ B_0 \end{pmatrix} = M_1 M_2 \dots M_N \begin{pmatrix} E_N \\ B_N \end{pmatrix} = M \begin{pmatrix} E_N \\ B_N \end{pmatrix} \tag{2}$$

where M_i is the transfer matrix of the ith layer, E_0 and B_0 are the electric and magnetic field amplitudes at the first interface, and E_N and B_N the field amplitudes at the last interface. Assuming that the incident wave is coming from the left with electric field amplitude normalized to a unitary value and also that one has only the outgoing transmitted wave at the right of the sample, the complex transmission coefficient of such sample can be written as

$$t(\omega) = \frac{2p_i}{(m_{11} + m_{12}p_o)p_i + (m_{21} + m_{22}p_o)} \tag{3}$$

where m_{ij}'s are the elements of the total transfer matrix M. p_i and p_o are related to the input and output media. All quantities computed here are related with the ratio between the amplitudes of the outgoing and incoming waves, which given the transmission $T(\omega) = \frac{p_o}{p_i}|t(\omega)|^2$.

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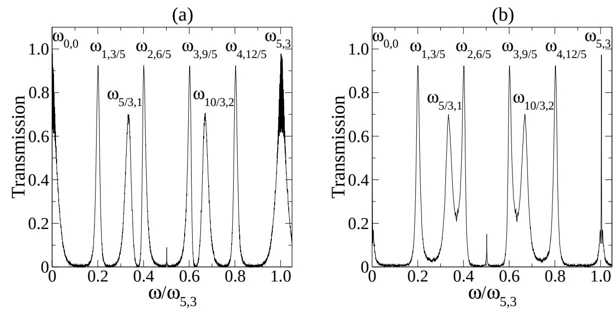

Fig. 1. Transmission spectrum of (a) an ordinary multilayered structure (composed uniquely of ordinary dielectric layers) and (b) a mixed one (composed of metamaterial and ordinary dielectric layers) for the particular case of $|n_a d_a| = \frac{5}{3}|n_b d_b|$. Both cases have a similar transmission peaks pattern. Peaks with finite transmission occur whenever a multiple of the mode half-wavelength equals the optical length of just one constituent layer. At the vicinity of the fully transparent resonant modes $\omega_{0,0}$ and $\omega_{5,3}$, the transmission band width is much narrower for the mixed case indicating a stronger finite-size scaling of these transmission peaks.

3. Results

For the numerical analysis, we assume non-absorbing dispersionless layers. The multilayer structure is composed of a random sequence of two distinct dielectric slabs, namely, an ordinary dielectric *A* and a left-handed dielectric *B*. The refraction indices and thicknesses were taken in such a way that the optical lengths on the slabs *A* and *B* were $|n_a d_a| = \frac{5}{3}|n_b d_b|$. If the slabs were chosen to satisfy the Bragg condition $|n_a d_a| = |n_b d_b|$, we would have the emergence of a *zero-*$\bar{n}$ *gap*. This kind of gap is insensitive to disorder [**?**]. This condition taken on a dispersionless multilayered structure creates a large photonic bang-gap that covers the whole spectrum, except for singular modes where the phase change $\delta = \omega n d/c = i\pi$ $(i = 0, 1, 2\ldots)$. This scenario does not support transmitting bands in finite-size structures. The disorder was implemented assuming that the i-*th* layer of the sequence has the same probability of being type *A* or *B*. In order to be able to explicitly show the direct influence of the metamaterial slabs, all quantities were obtained both for a mixed structure (consisting of matematerials and ordinary dielectric layers) as well as for an ordinary structure (consisting of only ordinary dielectric layers). In both cases, we assumed the same absolute values of refraction indices and thicknesses.

The first quantity analyzed was the transmission spectrum. Fig. 1 represents an average over 10^3 realizations of disorder in a structure with $N = 200$ layers. The plots are related to the ordinary structure Fig.1(a) and the mixed structure containing a metamaterial Fig.1(b). We observe the same pattern of transmission peaks irrespective to the presence or not of the metamaterial. These resonant modes occur when multiple integers of the half-wavelength equal the optical length of one of the constituent layers. Under this condition, the electromagnetic wave is not scattered by such layer, independently of the refraction index sign. We labeled the resonance frequencies as $\omega_{i,j}$, where i and j are associated with the number of half-wavelength fitting in the optical length of each layer. The index i is related to the larger optical length layer and j is related to the smaller one. If i and j are both integers, the mode becomes fully transparent. Formally, it reflects the fact that all matrices in the product (2) turn out to be $\pm I$, where I is the identity operator. Other resonant modes occur when just one of the indices is integer. In this case, only the matrices associated with the layers that fit integer numbers of half-wavelength become $\pm I$. In this case, the system behaves like a single slab composed of the other material.

An important feature is observed in the transmission spectrum around the fully transparent

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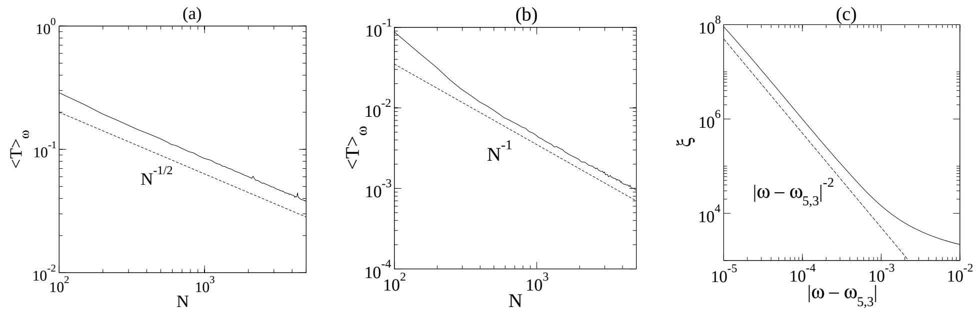

Fig. 2. (a) Spectral average of the transmission versus the number of layers N in a frequency range around the fully resonant mode of an ordinary structure. The transmission decays as $1/N^{1/2}$. (b) Same as before for a structure composed of ordinary and metamaterial layers. In this case, the finite-size dependence is stronger, with the transmission decaying asymptotically as $1/N$. (c) Localization length near the resonant mode $\omega_{5,3}$ of a mixed structure containing $N = 10^4$ layers. The quadratic divergence reproduces the same scaling law found in purely ordinary structures.

modes $\omega_{0,0}$ and $\omega_{5,3}$. In these frequency regions, the transmission band width is much narrower for the mixed structure containing the metamaterial. In order to investigate more accurately this fact, we compute the spectral average of the transmission in the frequency range around these peaks, as shown in Fig. 2(a-b). The result represents an average over 10^2 realizations of disorder. We compare the finite-size scaling behavior for the ordinary [Fig. 2(a)] and mixed [Fig. 2(b)] multilayer structures. The spectrally averaged transmission for the purely ordinary case decays as $1/N^{1/2}$. The same exponent was found in Ref. [18] where the layer parameters satisfied the Bragg condition. We found a new scaling law when one of the layers is composed of a metamaterial. In this case, the averaged transmission has a stronger dependence on the total number of layers, decaying asymptotically as $1/N$. Another important quantity to characterize the Anderson localization of light in disordered dielectric structures is the localization length (ξ), considered as the inverse of the Lyapunov exponent $[\xi = 1/\Lambda = -\lim_{N\to\infty}(N/\ln T)]$. Fig. 2(c) shows an average over 10 realizations of disorder, computed in the vicinity of $\omega_{5,3}$ mode. We observe the localization length diverging quadratically as one approaches the resonance. It reproduces the same scaling law previously reported for the ordinary case [18].

The interplay of the strong finite-size decay of the transmission and the stabilization of resonance necklace modes by disorder[15, 16, 19] suggests that the influence of disorder on the transmission properties may show new features in the presence of metamaterials. In order to explore this point, we investigate the spectral average of the transmission as a function of the disorder strength. Starting with an initial periodic structure of alternating A and B layers, the disorder strength was controlled by introducing the probability q of the i*th* layer to be replaced by the other species. For $q = 0$, one retains the periodic sequence, while the $q = 1/2$ limit recovers the uncorrelated fully disordered sequence. Our results are shown in Fig. 3 for $N = 10^2$ layers, averaged over 5×10^3 disorder configurations and within a spectral range around the $\omega_{5,3}$ mode. “O” labels the ordinary case and “M” corresponds to the mixed case.

The most prominent disorder effect is to promote the Anderson localization and, consequently, to reduce the transmission band. This phenomenon is due to the multiple incoherent superposition of waves scattered at the layers interfaces. This feature is clearly observed for the purely ordinary structure (see Fig. 3) on which the spectral average of the transmission decreases monotonically as the disorder strength increases. A similar trend is observed for the mixed structure in the weak disorder limit. However, for such mixed structure, the spectrally

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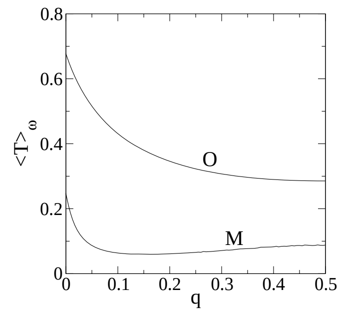

Fig. 3. Spectral and disorder averaged transmission in the vicinity of the fully transparent mode as a function of the disorder strength *q*. Average was performed over 5×10^3 disorder configurations in structures containing 10^2 layers. The mixed case (M) shows a minimum transmission at a finite disorder strength. Since the structure containing a metamaterial is more sensitive to disorder than the ordinary structure, it exhibits a smaller average transmission than the purely ordinary (O) structure

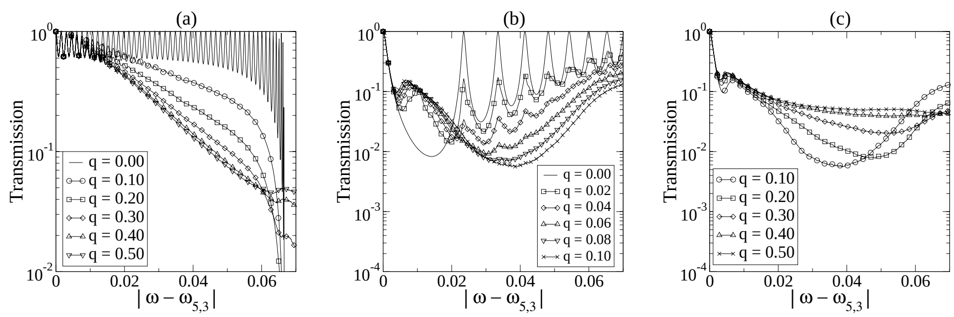

Fig. 4. Transmission spectra in the vicinity of the $\omega_{5,3}$ mode for different values of the disorder strength. We considered structures with $N=10^2$ layers and averaged over 10^5 realizations of disorder. Figure (a) shows the spectra of ordinary structures. In this case, the transmission monotonously decreases with the disorder strength. Figure (b) shows the weak disorder region of a mixed structure, where the predominant effect is the strong Anderson localization which leads to a faster decrease of the transmission as the disorder strength is increased. In the strong disorder region (c), the transmission of a mixed structure is enhanced by increasing disorder strengths as the number of resonant necklace modes also increases.

averaged transmission presents a minimum value around $q=0.15$. This unexpected behavior is related to a secondary disorder effect, which becomes more prominent in the mixed structure. Disorder also promotes the emergence of a few resonance necklace modes that may lead to a small transmission in frequency gaps. Such secondary effect is hidden by the slow development of the gap in ordinary structures, as shown in Fig. 4(a). In the mixed structure containing a metamaterial, the gap develops rather quickly due to the stronger finite-size scaling dependence of the average transmission. This leads to the strong decay of the transmission in the weak disorder regime reported in Fig. 4(b). At weak disorder, where the Anderson localization is predominant, the band structure of the periodic case ($q=0$) gives place to a wide low-transmission frequency range. The contribution of the resonant necklace states to the transmission thus becomes predominant in the high-disorder regime. Once these modes become more frequent at strong disorder, increasing disorder promotes an enhancement of the light transmission in the spectral range around the fully transparent mode, as seen in Fig. 4(c).

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Finally, we would like to stress that the narrow transmission peak in the close vicinity of the fully transparent mode is quite insensitive to disorder in the mixed structure, as can be seen in Fig.4(b-c). Actually, this peak resembles the singular frequency points of mixed structures with *zero*-$\bar{n}$ [12]. In finite structures, the singularity is replaced by a narrow peak whose width is inversely proportional to the total number of layers, which is consistent with our previous finding of a $1/N$ scaling law for the spectrally averaged transmission.

4. Summary and conclusions

In this work, we reported that the inclusion of metamaterials on the composition of a random binary multilayered structure changes the finite-size scaling law for the transmission in the frequency range around the fully transparent mode. The spectrally averaged transmission decays as $1/N$ which is faster than the scaling-law obtained for ordinary structures. In spite of this, the localization length obeys the ordinary quadratic scaling-law as the frequency approaches the resonance mode. The faster finite-size scaling and the emergence of a few transmission necklace modes inside the band-gap at the strong disorder regime promote a non-monotonous disorder dependence of the transmission in the frequency range around the fully transparent mode. This new feature is in contrast with the usual Anderson localization phenomenon that implies in a decreasing transmission as a function of the disorder strength. In structures containing metamaterials there is a characteristic disorder strength for which the system shows a minimum on the spectrally averaged transmission. Although we limited our investigation to non-absorbing structures, the main role played by absorption shall be to promote an overall reduction of the transmission in the vicinity of the resonant modes, with no direct impact on the finite-size scaling law or on the non-monotonicity of the spectrally averaged transmission. Our findings indicate that the strength of disorder shall be carefully chosen in the development of devices, such as band filters, that exploit the joint properties of binary structures, light localization and left-handed materials.

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