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Extended acoustic modes in random systems with *n*-mer short range correlations

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Abstract
In this paper we study the propagation of acoustic waves in a one-dimensional medium with a short range correlated elasticity distribution. In order to generate local correlations we consider a disordered binary distribution in which the effective elastic constants can take on only two values, η_A and η_B. We add an additional constraint that the η_A values appear only in finite segments of length n. This is a generalization of the well-known random-dimer model. By using an analytical procedure we demonstrate that the system displays $n-1$ resonances with frequencies ω_r. Furthermore, we apply a numerical transfer matrix formalism and a second-order finite-difference method to study in detail the waves that propagate in the chain. Our results indicate that all the modes with $\omega \neq \omega_r$ decay and the medium transmits only the frequencies ω_r.

1. Introduction

The localization theory proposed by Anderson has predicted the absence of extended eigenstates in low-dimensional systems with uncorrelated disorder [1]. Therefore, for long times, the width of the time-dependent wavepacket saturates in a finite region around the initial position. In a three-dimensional lattice, the presence of weak disorder promotes the localization of the high energy eigenmodes [1, 2]. The low energy states with long wavelength remain extended, although acquiring a finite coherence length. A mobility edge separates the high energy localized states from the low energy extended states [1, 2]. Recently, it has been shown that low-dimensional disordered systems can support extended states or a localization–delocalization transition in the presence of short or long range correlations in the disorder distribution [3–15]. From the experimental point of view, these theoretical predictions have been useful for explaining transport properties of semiconductor superlattices [16] and microwave transmission spectra of a single-mode waveguide with intentional correlated disorder [17].

The formalism of the localization theory applies also to the study of magnon localization in random ferromagnets [18, 19], collective vibrational motion of one-dimensional (1D) disordered harmonic chains [20–22] and acoustic waves in disordered media [23–35]. In fact, the propagation of acoustic waves has attracted both theoretical [23–31, 33–35] and experimental [32] interest. Along general lines, it was shown that such waves may be localized in media with uncorrelated disorder. However, recent works point out the drastic effect of correlations within the acoustic wave context [27–31, 33–35]. In [27] the propagation of acoustic waves in the random-dimer chain was studied using the transfer matrix method, exact analytical analysis, and direct numerical simulation of the scalar wave equation. The results indicated that there exists a resonance frequency at which the localization length of the acoustic wave diverges [27]. It was also shown that only the resonance frequency can propagate through the 1D media. Moreover, the wave propagation in a random system with a power-law correlation function was investigated by using the renormalization group formalism as well as numerical methods [28–31]. Calculations indicate that there can be a disorder-induced transition from delocalized to localized states of acoustic waves in any spatial dimension. In [33] the propagation of acoustic waves in a 1D medium with the random elasticity distribution assumed to have a power spectrum $S(k) \sim 1/k^{\alpha}$ was studied. It was numerically demonstrated that scale-free correlations promote a stable phase of free acoustic waves in the thermodynamic limit for $\alpha > 2$ [33].

In this paper, we report further progress along these lines. Our main aim is to focus on the influence of short range correlated disorder on the propagation of elastic waves. In order to generate local correlations we will consider a disordered binary distribution in which the effective elastic constants can take on only two values, η_A and η_B. We add an additional constraint that the η_A values appear only in finite segments of length n. The n-mer correlated disorder is a generalization of the well-known random-dimer disorder [22, 27]. By using analytical and numerical formalisms, we will demonstrate that an elastic medium with n-mer correlations on the random elasticity distribution can support $n-1$ extended acoustic modes.

2. The model and formalism

We start by considering the acoustic wave equation in a random medium [27]:

$$\frac{\partial^2}{\partial t^2}\psi(x,t) = \frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x,t)}{\partial x}\right]. \tag{1}$$

Here $\psi(x,t)$ is the wave amplitude, t is the time, and $\eta(x) = e(x)/m$ is the ratio of the stiffness $e(x)$ and the medium mean density m. We consider the wave amplitude with a time-dependent harmonic form $\psi(x,t) = \psi(x)\exp(-\mathrm{i}\omega t)$, where ω is the wave frequency. We will use a finite-difference (FD) method to write the acoustic wave equation in a discretized form. The spatial wave amplitude $\psi(x)$ is written as ψ_i, where $x = \mathrm{i}\Delta x$. The spatial derivative will be written as $(\partial\psi(x))/(\partial x) \approx (\psi_i - \psi_{i-1})/\Delta x$. Following [27] we will use $m = 1$ and consider nearest-neighbor spacing $\Delta x = 1$. Therefore, the right side of equation (1) can be written as

$$\frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x)}{\partial x}\right] \approx [\eta_i(\psi_{i+1}-\psi_i) - \eta_{i-1}(\psi_i - \psi_{i-1})]. \tag{2}$$

Accordingly, the discrete 1D version of the wave equation can be obtained as

$$\eta_i(\psi_{i+1}-\psi_i) - \eta_{i-1}(\psi_i-\psi_{i-1}) + \omega^2\psi_i = 0. \tag{3}$$

The elastic constants η_i will be generated following an n-mer distribution [22]. We introduce a disordered binary distribution in which the elastic constants η_i can take on only two values, η_A and η_B, with probabilities p and $1-p$ respectively. We add an additional constraint that the η_A values appear only in finite segments of length n. This is a generalization of the random-dimer model, where $n = 2$ recovers the well-known random-dimer elastic system form [22, 27].

3. Results

3.1. Analytical analysis

To study the nature of acoustic modes in n-mer correlated elastic media we will follow the analytical formalism used in [22, 27]. Equation (3) can be solved by using the transfer matrix formalism (TMF) [20, 27]. The TMF is obtained from a matrix recursive reformulation of equation (3). The matricial equation is

$$\begin{pmatrix}\psi_{i+1}\\ \psi_i\end{pmatrix} = \begin{pmatrix}\frac{-\omega^2+\eta_i+\eta_{i-1}}{\eta_i} & -\frac{\eta_{i-1}}{\eta_i}\\ 1 & 0\end{pmatrix}\begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix} = T_i\begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix}. \tag{4}$$

The wave amplitude of the complete 1D system is given by the product of the transfer matrices $Q_N = \prod_{i=1}^N T_i$. Within the n-mer distribution of elastic constants η_A and η_B we will have four different types of transfer matrix T_i, namely [27] T_{AA}, T_{AB}, T_{BA} and T_{BB}, where

$$T_{AA} = \begin{pmatrix}\frac{-\omega^2+2\eta_A}{\eta_A} & -1\\ 1 & 0\end{pmatrix} \tag{5}$$

and

$$T_{AB} = \begin{pmatrix}\frac{-\omega^2+\eta_A+\eta_B}{\eta_B} & -\frac{\eta_A}{\eta_B}\\ 1 & 0\end{pmatrix}, \tag{6}$$

while the other two are obtained by carrying out the transformations $A \to B$ and $B \to A$. Following [27], the n-mer short range correlated elastic medium displays extended acoustic modes for $T_{AA}^n = -p_{n-2}(x_\mathrm{r})I$ and $T_{BA}T_{AA}^{n-1}T_{AB} = p_{n-2}(x_\mathrm{r})T_{BB}$, where $x_\mathrm{r} = (2\eta_A - \omega_\mathrm{r}^2)/\eta_A$ are the solutions of the $n-1$th-order polynomial equation $p_{n-1}(x_\mathrm{r}) = 0$. The characteristic polynomials can be obtained following the recursive relations $p_{n+1}(x) = xp_n(x) - p_{n-1}(x)$ with $p_1(x) = x$ and $p_0(x) = 1$. Thus, for these frequencies ω_r the transfer matrix Q_N of the complete chain contains only the matrices T_{BB}, which effectively describe an ordered chain. For example, following the formalism above we obtain the frequency $\omega_\mathrm{r} = \sqrt{2\eta_A}$ for $n = 2$; $\omega_\mathrm{r}^1 = \sqrt{\eta_A}$ and $\omega_\mathrm{r}^2 = \sqrt{3\eta_A}$ for $n = 3$; $\omega_\mathrm{r}^1 = \sqrt{(2-\sqrt{2})\eta_A}$, $\omega_\mathrm{r}^2 = \sqrt{2\eta_A}$ and $\omega_\mathrm{r}^3 = \sqrt{(2+\sqrt{2})\eta_A}$ for $n = 4$; $\omega_\mathrm{r}^1 = \sqrt{(2-\sqrt{\frac{3+\sqrt{5}}{2}})\eta_A}$, $\omega_\mathrm{r}^2 = \sqrt{(2-\sqrt{\frac{3-\sqrt{5}}{2}})\eta_A}$, $\omega_\mathrm{r}^3 = \sqrt{(2+\sqrt{\frac{3-\sqrt{5}}{2}})\eta_A}$ and $\omega_\mathrm{r}^4 = \sqrt{(2+\sqrt{\frac{3+\sqrt{5}}{2}})\eta_A}$ for $n = 5$. Therefore, disordered elastic media with n-mer-like short range correlations display $n-1$ extended states in the thermodynamic limit.

3.2. Numerical analysis

The logarithm of the smallest eigenvalues of the limiting matrix $\Gamma = \lim_{N\to\infty}(Q_N^\dagger Q_N)^{1/2N}$ defines the Lyapunov exponent γ (the inverse of localization length $\lambda = 1/\gamma$). Further details about the computation of this parameter can be found in [2, 27]. For extended states, $\lambda/N \approx$ const and it goes to zero for localized waves. Typically, we use up to $N = 10^7$ transfer matrices to compute the localization

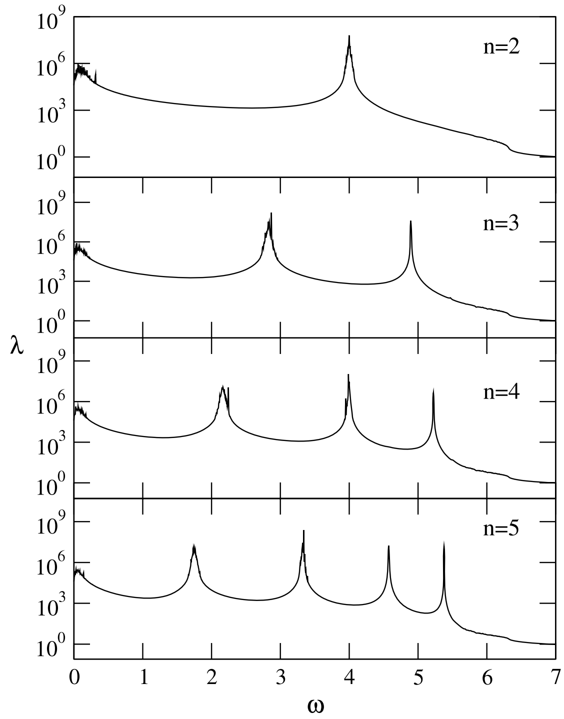

Figure 1. Numerical calculations of the localization length versus acoustic mode frequency ω. Calculations were done for a very long 1D ($N \approx 10^7$ sites) n-mer correlated elastic medium with $\eta_A = 8$, $\eta_B = 10$, $n = 2$–5. We demonstrated, in perfect agreement with analytical calculations, the existence of $n - 1$ resonances.

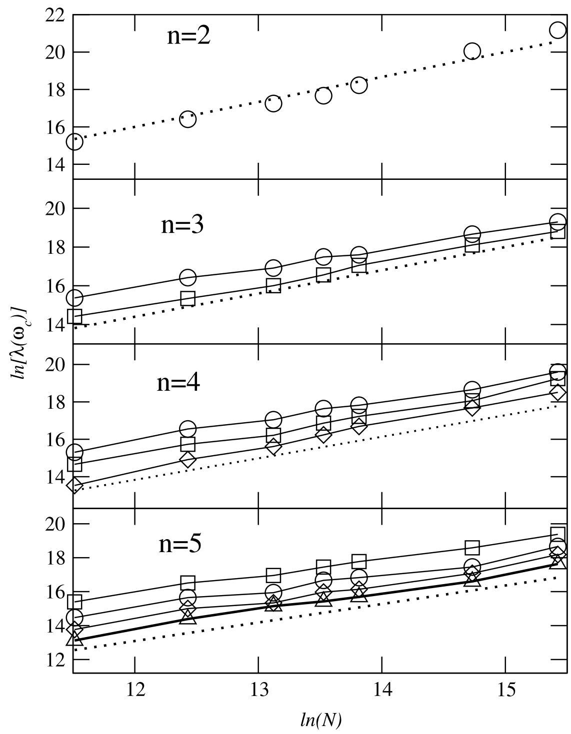

Figure 2. $\ln \lambda(\omega = \omega_r) \times \ln N$ for an n-mer 1D correlated elastic medium with $\eta_A = 8$, $\eta_B = 10$, $n = 2$–5. Dotted lines represent the extended behavior $\lambda \propto N$. Our calculations reveal, in good agreement with analytical treatment, the extended nature of resonance in the n-mer 1D correlated elastic media.

length. It should be stressed that in this method, self-averaging effects automatically take care of statistical fluctuations. The resulting data set has statistical errors of less than 5%. We estimate and control these statistical fluctuations following the deviations of the calculated eigenvalues of two adjacent iterations [2, 27]. All numerical calculations were done for $p = 0.5$; however, we did not find any dependence on p within the open interval (0, 1). In figure 1 we plot $\lambda \times \omega$ for an n-mer 1D correlated elastic medium with $\eta_A = 8$, $\eta_B = 10$, $n = 2$–5. Our results indicate, in good agreement with analytical calculations, the presence of $n-1$ resonances in the high frequency region. The frequencies ω_r of these resonances agree remarkably well with the frequencies obtained through the analytic formalism. In figure 2 we plot a finite size scaling of the localization length at frequencies ω_r. We perform calculations of $\ln \lambda(\omega = \omega_r) \times \ln N$ considering the same parameters as for figure 1. Dotted lines represent the extended behavior $\lambda \propto N$. Our results indicate that the localization length diverges linearly with system size for all frequencies, thus confirming numerically the extended nature predicted earlier. Around the critical frequencies the localization length diverges as the frequency approaches the critical frequency as $\lambda \propto |\omega - \omega_r|^{-\nu}$. Using the logarithmic plot of $\lambda \times (\omega - \omega_r)$ we estimated the exponent ν. Calculations are shown in figure 3. For the resonant modes considered here ($n = 2$–5), λ diverges as $\lambda \propto |\omega - \omega_r|^{-2.00(5)}$. We stress that this divergence is exactly the same as that found in [27] and seems to be a signature of dimer-like local correlations [3, 4]. In addition, we apply the FD method with second-order discretization for both time and spatial variables proposed in [27]. Thus, in discretized form, $\psi(x, t)$ is written as ψ_i^j, where j denotes the time step number and i is the grid point number [27]. Therefore, the second time derivative in equation (1) is given by [27]

$$\frac{\partial^2}{\partial t^2}\psi(x,t) \approx \frac{\psi_i^{j+1} - 2\psi_i^j + \psi_i^{j-1}}{\Delta t^2}, \tag{7}$$

where Δt is the size of the time step. The spatial derivative will be written as

$$\frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x,t)}{\partial x}\right] \approx \frac{1}{\Delta x^2}[\eta_i(\psi_{i+1}^j - \psi_i^j) - \eta_{i-1}(\psi_i^j - \psi_{i-1}^j)]. \tag{8}$$

In our calculations the spacing Δx between two neighboring grid points was set to $\Delta x = 1$. In order to ensure the stability of the discretized equations we will use $\Delta t < \Delta x/20$. We carry out our dynamical analysis by sending a wave from one side of the chain ($L = 0$) and recording the transmitted wave close to the other side (position $L = 20\,000$). We calculate the intensity spectrum of the transmitted wave at position L defined as

$$A(\omega) = (1/2)|\psi_L(\omega)|^2 \tag{9}$$

where $\psi_L(\omega)$ is the Fourier transform of the transmitted wave $\psi_L(t)$ at position $L = 20\,000$. For transmitted acoustic modes, $A(\omega) > 0$ and goes to zero for filtered ones. In our dynamical calculations the chain length was $N = 30\,000$. In figure 4 we

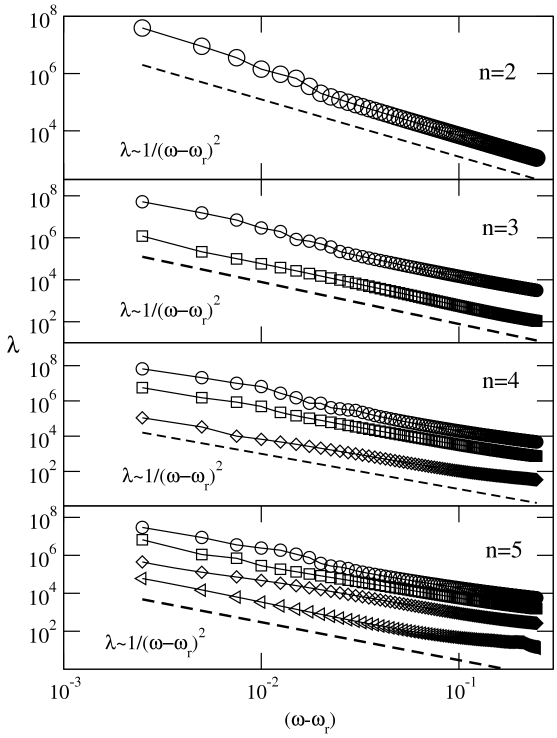

Figure 3. Logarithmic plot of λ × $(\omega - \omega_r)$. Calculations were done for the same parameters as were used in figures 1 and 2. We found that for the resonant modes considered here (n = 2–5), λ diverges as $\lambda \propto |\omega - \omega_r|^{-2.00(5)}$.

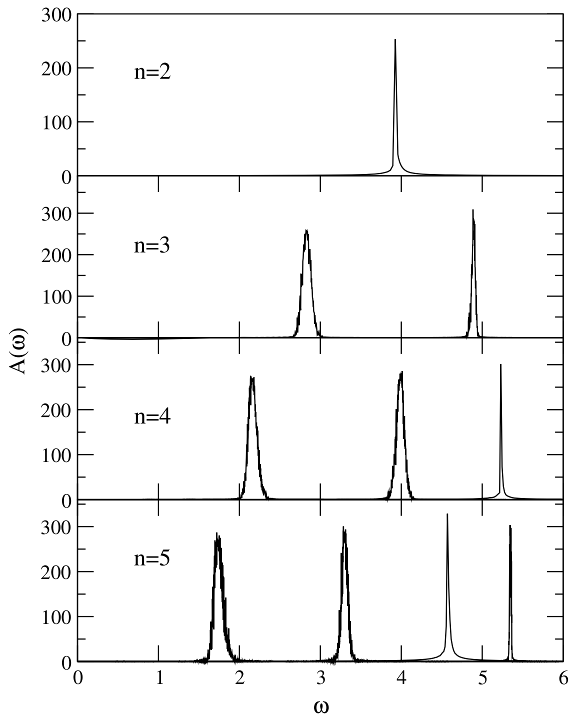

Figure 4. Intensity spectrum of the transmitted wave pulse through the n-mer 1D correlated elastic medium with $\eta_A = 8$, $\eta_B = 10$, n = 2–5. Corroborating analytical and numerical results shown previously, all the modes with $\omega \neq \omega_r$ decay and the medium transmits only the frequency ω_r.

present the resulting frequency dependence of the intensity spectrum $A(\omega)$ for our dynamical numerical analysis. We use the incident pulse $\Psi_0(t) = \exp[-(t - t_0)^2/2\sigma_t^2]\cos(\omega t)$ with $\sigma_t = (1/\sigma_\omega) = 1/20$ [27]. Calculations were done for an n-mer 1D correlated elastic medium with $\eta_A = 8$, $\eta_B = 10$, n = 2–5. We averaged the intensity spectrum using 30 realizations of the disorder. In good agreement with the analytical and numerical formalism shown previously, all the modes with $\omega \neq \omega_r$ decay and the medium transmits only the frequency ω_r.

4. Summary and conclusions

In this paper we have considered the propagation of acoustic waves in a 1D elastic medium with short range correlated disorder. The elasticity distribution was considered as an n-mer dimer distribution of effective spring constants η_A and η_B. Here the η_A value appears only in finite segments of length n. We applied both analytical and numerical procedures to study the nature of acoustic modes that propagate along the medium. We carried out an exact analysis of the effective transfer matrix in order to predict the existence of extended acoustic modes and the dependence of the resonance frequencies ω_r on the value of the paired elastic constant and the degree of short range correction. Our results indicate that the n-mer model supports $n - 1$ extended states with $\lambda(\omega_r) \propto N$. Analytical results were corroborated by numerical estimation of the localization length λ. Furthermore, we numerically demonstrated that around the critical frequencies the localization length diverges as $\lambda \propto |\omega - \omega_r|^{-2.00(5)}$. Our calculations indicated that the quadratic divergence found here seems to be a universal feature associated with local n-mer correlations [27]. In addition, by solving the scalar wave equation for propagation of a incident pulse with a wide spectral density, we showed that the chain localizes all the frequency content of the wave pulse, except for the resonance frequencies. Within the context of recent works on acoustic waves in low-dimensional media with correlated disorder [28–31], our results extend some previous statements concerning the existence of resonant modes in 1D elastic media with dimer-like correlated disorder. We demonstrated the existence of a multiple-resonance structure in an elastic medium with n-mer correlated elasticity distribution and showed that these resonances have specificities similar to those found in 1D elastic media with dimer-like correlated disorder. We expect that the present work will stimulate further theoretical and experimental investigations along these lines.

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