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Absence of localized acoustic waves in a scale-free correlated random system

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Abstract
We numerically study the propagation of acoustic waves in a one-dimensional medium with a scale-free long-range correlated elasticity distribution. The random elasticity distribution is assumed to have a power spectrum $S(k) \sim 1/k^{\alpha}$. By using a transfer-matrix method we solve the discrete version of the scalar wave equation and compute the localization length. In addition, we apply a second-order finite-difference method for both the time and spatial variables and study the nature of the waves that propagate in the chain. Our numerical data indicate the presence of extended acoustic waves for a high degree of correlations. In contrast with local correlations, we numerically demonstrate that scale-free correlations promote a stable phase of free acoustic waves in the thermodynamic limit.

1. Introduction

The absence of extended eigenstates in low-dimensional systems with uncorrelated disorder was pointed out by Anderson using perturbation theory and scaling analysis [1, 2]. Therefore, after a long time the width of the time-dependent wavepacket saturates in a finite region around the initial position. In a three-dimensional lattice, the presence of weak disorder promotes the localization of the high-energy eigenmodes [1, 2]. The low-energy states with long wavelength remain extended, although acquiring a finite coherence length. A mobility edge separates the high energy localized from the low energy extended states [1, 2]. Recently, it has been shown that low-dimensional disordered systems can support extended states or a localization–delocalization transition in the presence of short- or long-range correlations in the disorder distribution [3–15]. The delocalization problem in one-dimensional (1D) systems with scale-free correlated diagonal disorder has attracted much attention. It has been reported [5, 9, 10, 13] that these systems display an Anderson metal–insulator transition (MIT) with mobility edges separating localized and extended states for sufficiently strong correlations. In particular, the 1D system with nearest-neighbor interactions and a long-range correlated on-site disorder distribution with a power-like spectrum behaving as $k^{-\alpha}$ has been studied in detail in [5, 10, 13]. From the experimental point of view, these theoretical predictions were useful to explain the transport properties of semiconductor superlattices [16] and microwave transmission spectra of a single-mode waveguide with intentional correlated disorder [17]. Moreover, it was suggested that an appropriate algorithm for generating random correlated sequences with desired mobility edges could be used in the manufacture of filters for electronic or optical signals [9].

The localization of collective excitation in random low-dimensional lattices is a quite general feature. It applies, for example, to the study of magnon localization in random ferromagnets [6], collective vibrational motion of 1D disordered harmonic chains [7, 18], and acoustic waves in disordered media [19–28]. In fact, the propagation of acoustic waves has attracted both theoretical [19–27] and experimental [28] interest. In general terms, it was shown that such waves may be localized in media with uncorrelated disorder. However, recent works point out the drastic effect of correlations within the acoustic waves context [23–27]. In [23] the propagation of acoustic waves in the random-dimer chain was studied using the transfer-matrix method, exact analytical analysis, and direct numerical simulation of the scalar wave equation. The results indicated that there exists a resonance frequency at which the localization length of the acoustic wave diverges [23]. It was also shown that only the resonance frequency can propagate through the 1D medium. Moreover, the wave propagation in a random system with a power-law correlation function was investigated by using renormalization group formalism as well as numerical methods [24–27]. Calculations indicate that there can be a disorder-induced transition from delocalized to localized states of acoustic waves in any spatial dimension.

In this work, we contribute to a further understanding of acoustic wave propagation in low-dimensional systems with correlated disorder distribution. We follow [23] considering a discrete 1D version of the wave equation where the elasticity distribution appears as an effective spring constant. The scale-free long-range correlated elastic constant distribution was generated by using a discrete Fourier method. First, using the transfer-matrix method, we calculate the localization length of acoustic waves propagating in the medium, and show that long-range correlation induces a localization–delocalization transition. In addition, by using direct numerical simulation of the equation that governs the propagation of acoustic waves, we demonstrated the drastic effect of free scale correlations within the disorder distribution. We find that the correlated random medium can filter out all high frequencies of the wavepacket.

2. Model and formalism

We start by considering the acoustic wave equation in a random medium (see [23]):

$$\frac{\partial^2}{\partial t^2}\psi(x,t) = \frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x,t)}{\partial x}\right]. \tag{1}$$

Here, $\psi(x,t)$ is the wave amplitude, t is the time, and $\eta(x) = e(x)/m$ is the ratio of the stiffness $e(x)$ and the medium's mean density m. Following [23] we will use $m = 1$ and consider a discrete 1D version of the wave equation ($\Delta x = 1$)

$$\eta_i(\psi_{i+1} - \psi_i) - \eta_{i-1}(\psi_i - \psi_{i-1}) + \omega^2\psi_i = 0. \tag{2}$$

The elastic constants η_i will be considered as a long-range correlated random sequence. In order to generate sequences with a power-law decaying spectral density function, we firstly generate the following auxiliary sequence [5, 29]:

$$x_i = \tanh\left[\sum_{k=1}^{N/2}\frac{1}{k^{\alpha/2}}\cos\left(\frac{2\pi i k}{N} + \phi_k\right)\right] \tag{3}$$

which is restricted to the interval $-1 \leqslant x_i \leqslant 1$ and whose spectral density function decays asymptotically as $1/k^\alpha$. The hyperbolic transformation of the series brings the advantage of bounding the interval of the random variable without changing its asymptotic correlation function. Such a power-law decaying correlation function actually characterizes the absence of a typical correlation length in the disorder distribution and allows the investigation of the influence of scale-free disorder on the properties of the acoustic waves. In the above equation, k is the wavevector of the modulations on the random variable landscape, ϕ_k are $N/2$ random phases uniformly distributed in the interval $[0, 2\pi]$ and the exponent α controls the degree of correlation. The sequence of elastic constants is obtained after normalizing the auxiliary sequence to have unitary variance ($\Delta\eta = 1$) and displacing it to avoid negative constants η_i. In the following, we use $\eta_i = 2 + x_i/\Delta x$. With the above procedure, the distribution of η_i has sharp edges for any value of α, which results in long-range correlated sequences of strictly positive elastic constants even when very large chains are considered. For $\alpha = 0$, we recover an uncorrelated random sequence of elastic constants.

2.1. Localization properties

Equation (2) can be solved by using the transfer-matrix formalism (TMF) [7, 23]. The TMF is obtained from a matrix recursive reformulation of equation (2). The matricial equation is

$$\begin{pmatrix}\psi_{i+1}\\ \psi_i\end{pmatrix} = \begin{pmatrix}\frac{-\omega^2+\eta_i+\eta_{i-1}}{\eta_i} & -\frac{\eta_{i-1}}{\eta_i}\\ 1 & 0\end{pmatrix}\begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix} = T_i\begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix}. \tag{4}$$

The wave amplitude of the complete 1D system is given by the product of the transfer matrices $Q_N = \prod_{i=1}^{N} T_i$. The logarithm of the smallest eigenvalues of the limiting matrix $\Gamma = \lim_{N\to\infty}(Q_N^\dagger Q_N)^{1/2N}$ define the Lyapunov exponent γ (inverse of localization length $\lambda = 1/\gamma$). Further details about the computation of this parameter can be found in [2, 23]. Typically, by using a fast Fourier formalism to sum equation (3), we use up to $N = 2^{22}$ transfer matrices to compute the localization length. For extended states $\lambda/N \approx$ const and goes to zero for localized waves. A quantitative scaling analysis of the localization number can be derived by using the scaled average localization length $\langle\lambda\rangle/N$ defined as

$$\langle\lambda\rangle/N = \frac{1}{NN_f}\sum_{\omega=0.5}^{\omega=1.5}\lambda(\omega) \tag{5}$$

where N_f is the number of acoustic modes within the interval [0.5, 1.5]. To compute the scaled average localization length, the bottom of the band was avoided because the localization lengths of these low-frequency modes are large even in the absence of correlated disorder [23]. We are interested in the existence of extended states apart from the bottom of the band. Accordingly, $\langle\lambda\rangle/N$ does not depend on N for extended modes and goes to zero for localized ones.

2.2. Dynamics of acoustic waves

In addition, we apply the finite-difference method with second-order discretization for both time and spatial variables proposed in [23]. Thus, in discretized form, $\psi(x,t)$ is written as ψ_i^n, where n denotes the time step number and i is the grid point number [23]. Therefore, the second time derivative in equation (1) is given by [23]

$$\frac{\partial^2}{\partial t^2}\psi(x,t) \approx \frac{\psi_i^{n+1} - 2\psi_i^n + \psi_i^{n-1}}{\Delta t^2} \tag{6}$$

where Δt is the size of the time step. The spatial derivative will be written as

$$\frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x,t)}{\partial x}\right] \approx \frac{1}{\Delta x^2} \times [\eta_i(\psi_{i+1}^n - \psi_i^n) - \eta_{i-1}(\psi_i^n - \psi_{i-1}^n)]. \tag{7}$$

In our calculations the spacing Δx between two neighboring grid points was set at $\Delta x = 1$. In order to ensure the stability of the discretized equations we will use $\Delta t < \Delta x/100$. We carry out our dynamical analysis by sending a wave from one side of the chain ($L = 0$) and recording the transmitted wave close to the other side (position $L = 20\,000$). We calculate

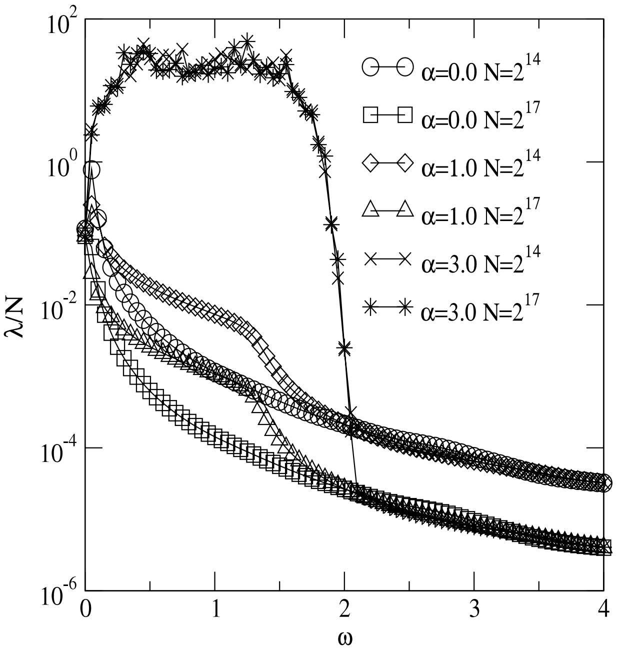

Figure 1. Scaled localization length λ/N versus ω for $\alpha = 0$, 1, and 3. Calculations were made considering $N = 2^{14}$ and 2^{17} points. These results indicate that, for strong correlations, there are extended acoustic waves at the low-frequency region.

the intensity spectrum of the transmitted wave at position L defined as

$$A(\omega) = (1/2)|\psi_L(\omega)|^2 \quad (8)$$

where $\psi_L(\omega)$ is the Fourier transform of the transmitted wave $\psi_L(t)$ at position $L = 20\,000$. For transmitted acoustic modes, $A(\omega) > 0$ and goes to zero for filtered ones. In our dynamical calculations the chain length was $N = 2^{15}$.

3. Results and discussion

Initially, we show the results about the localization properties obtained using the transfer-matrix technique. The finally obtained data have statistical errors less than 5%. We estimate and control these statistical fluctuations following the deviations of the calculated eigenvalues of two adjacent iterations [2, 23]. In figure 1 we show the scaled localization length λ/N versus ω computed for $\alpha = 0$, 1, 3, and distinct system sizes ($N = 2^{14}$ and 2^{17}). All calculations were averaged over 10^5 disorder configurations. For $\alpha = 0$ and 1 the localization length scales proportionally to the system size only for $\omega = 0$. Therefore, for $\omega > 0$ there are no truly delocalized states at this regime of weakly correlated disorder. However, for $\alpha = 3$ a well defined data collapse in a wide region of low frequencies is obtained with $\lambda \propto N$. This result suggests the possibility of a phase of low-frequency extended states for strongly correlated disorder. In figure 2(a) we collect data of the scaled average localization length $\langle\lambda\rangle/N$ versus the degree of correlations α for $N = 2^{14}$, 2^{17} and 2^{19}. Let us stress that to compute the average localization length the bottom of the band was avoided due to the weak localization character of these low-frequency acoustic modes

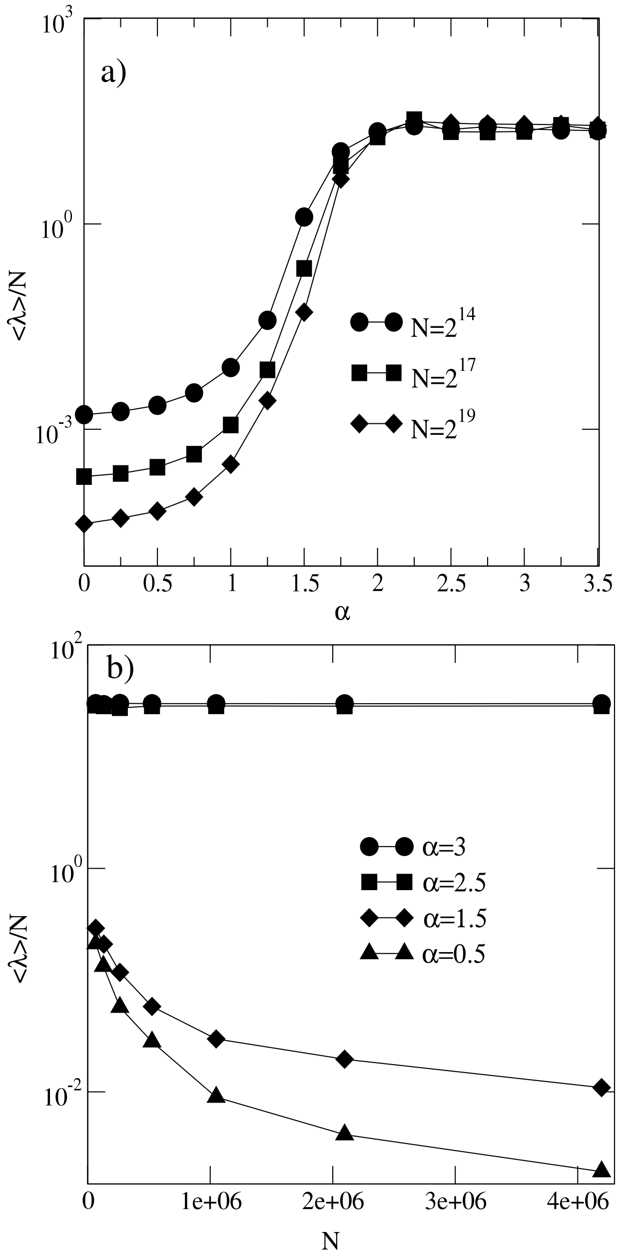

Figure 2. (a) Scaled average localization length $\langle\lambda\rangle/N$ versus the degree of correlations α. For $\alpha > 2$ there is a well defined data collapse, thus indicating a localized–delocalized transition. (b) Finite size scaling of the scaled average localization length $\langle\lambda\rangle/N$. Within our numerical precision $\langle\lambda\rangle \propto N^{0.98(2)}$ for $\alpha > 2$ thus indicating extended states.

even in the absence of correlated disorder [23]. As can be seen in figure 2(a), there is a well defined data collapse in the strongly correlated regime ($\alpha > 2$), i.e. the localization length diverges in the regime of low frequencies ($\omega < \omega_c \approx 1.6(1)$). In figure 2(b) we plot the scaled average localization length $\langle\lambda\rangle/N$ versus N for $N = 2^{14}$ up to 2^{22} and $\alpha = 0.5, 1.5, 2.5$ and 3. Within our numerical precision $\langle\lambda\rangle \propto N^{0.98(2)}$ for $\alpha > 2$. For $\alpha < 2$ the vanishing of the scaled average localization length $\langle\lambda\rangle/N$ for large N confirms the localized nature of the eigenstates in this regime. Therefore, the finite size scaling of the scaled average localization length $\langle\lambda\rangle/N$

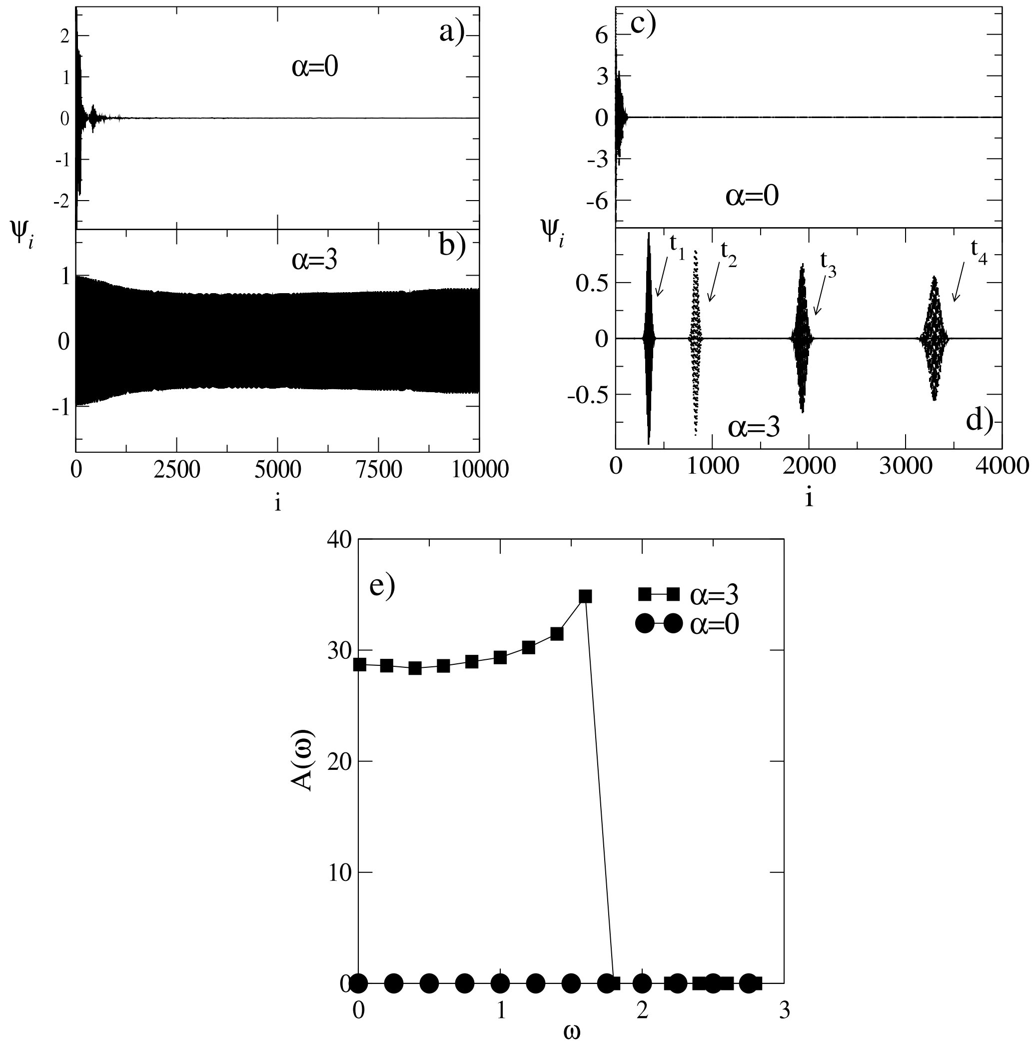

Figure 3. (a), (b) The amplitude of the wave during propagation through the scale-free correlated disordered medium for time $t = 500\,000\Delta t$. We consider in (a) the uncorrelated case ($\alpha = 0$) and (b) the strongly correlated limit ($\alpha = 3$). The incident wave is a sine wave with frequencies $\omega = 1$ (below ω_c). (c), (d) The amplitude of the wave for times $t_1 = 50\,000\Delta t$, $t_2 = 100\,000\Delta t$, $t_3 = 200\,000\Delta t$ and $t_4 = 300\,000\Delta t$, considering the incident wave as a pulse defined by $\Psi_0(t) = \exp[-(t - t_0)^2/2\sigma_t^2]\cos(\omega t)$ with $\sigma_t = (1/\sigma_\omega) = 20$ and frequency $\omega = 1$ (below ω_c). Regardless of the initial condition, the case $\alpha = 3$ allows the propagation along the 1D system. (e) The intensity spectrum $A(\omega)$ of the transmitted wave pulse at position $L = 20\,000$ computed using 20 realizations of the disorder. The incident wave was considered as a pulse defined by $\Psi_0(t) = \exp[-(t - t_0)^2/2\sigma_t^2]\cos(\omega t)$ with $\sigma_t = (1/\sigma_\omega) = 20$ and frequencies chosen within the interval $\{0, 3\}$. For sufficient degree of correlations, $A(\omega < \omega_c) > 0$, indicating that those acoustics waves with divergent localization lengths display a free propagation through the scale-free correlated disordered medium.

indicates the existence of a localized–delocalized transition for $\alpha > 2$. To conclude we will look at the evidence of the above phase transition by solving directly the scalar wave equation. By following the time propagation of an incident wave we obtain directly the degree of transmittance of a scale-free correlated disordered medium. Moreover, the divergence of the localization length itself does not guarantee the existence of extended states, as in the case of a vibrational wave envelope displaying a power-law decay [2]. In figures 3(a) and (b) we plot the wave amplitude ψ_i versus grid index i at time $t = 5000\,000\Delta t$. The incident wave is a sine wave with frequency $\omega_0 < \omega_c$. We consider in (a) the uncorrelated case ($\alpha = 0$) and (b) the strong correlated limit ($\alpha = 3$). We observe that for $\alpha = 3$ the incident wave displays a free propagation through the scale-free correlated disordered medium. By following [24, 25] we also consider the incident wave as a pulse defined by $\Psi_0(t) = \exp[-(t - t_0)^2/2\sigma_t^2]\cos(\omega t)$ with $\sigma_t = (1/\sigma_\omega) = 20$. In figures 3(c) and (d) we plot the wave amplitude ψ_i versus grid index i at times $t_1 = 50\,000\Delta t$, $t_2 = 100\,000\Delta t$, $t_3 = 200\,000\Delta t$, $t_4 = 300\,000\Delta t$ with $\sigma_t = (1/\sigma_\omega) = 20$, frequency $\omega = 1$ (below ω_c), and $\alpha = 0$ and 3 (respectively figures 3(c) and (d)). Once again, we observe that for $\alpha = 3$ the incident wave displays a free propagation through the scale-free correlated disordered medium. To complete our dynamical analysis, we solve numerically the wave equation for several pulses with distinct frequencies within the interval $\{0, 3\}$ and compute the intensity spectrum $A(\omega)$ using 20 realizations of the disorder. In figure 3(e) we present the resulting frequency dependence of the intensity spectrum $A(\omega)$ for these simulations. As

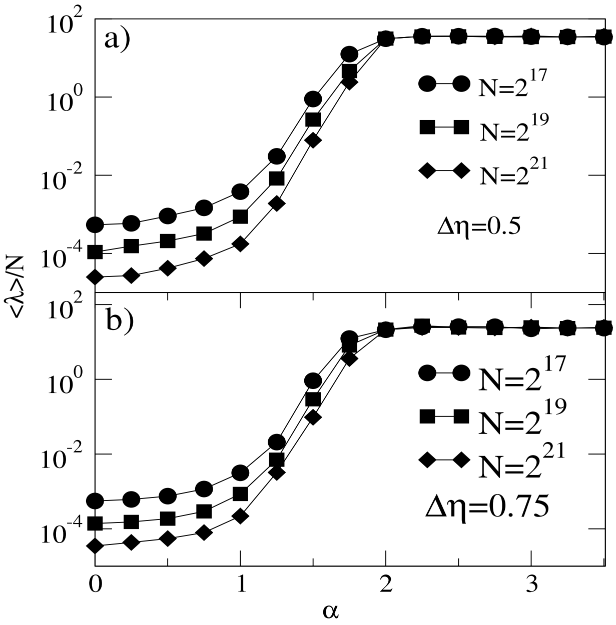

Figure 4. (a), (b) Scaled localization lengths, averaged over a frequency window [0.5, 1.5], versus α for $\Delta\eta = 0.5$ and 0.75. Similarly to the 1D Anderson model with long-range correlated diagonal disorder [13], the critical point (α_c) obtained here seems to be independent of the magnitude of disorder $\Delta\eta$.

shown in figure 3(e), all the modes with $\omega > \omega_c$ decay, and the medium behaves as a filter to transmit only the modes below frequency $\omega_c \approx 1.6$. We compute the intensity spectrum $A(\omega)$ by using another kind of incident wave (e.g. $\Psi_0(t) = \sum_{\omega_n<3} \cos(\omega_n t)$) and no qualitative change in the physical properties is found. These results confirm those obtained by the numerical analysis based on the TMF method described before. Then the numerical evidence reported here, obtained by using TMF and numerical solutions of wave equations, suggests that the low-frequency modes in a 1D medium with scale-free correlated disorder are in fact delocalized. The localization–delocalization transition found here is similar to the electronic Anderson transition induced by long-range correlations found previously in 1D random electronic systems [5]. Before finishing, we explore the possibility of the disorder strength to influence the critical value $\alpha_c = 2$. In figures 4(a) and (b) we show the scaled average localization length $\langle\lambda\rangle/N$ versus the degree of correlation α for $N = 2^{17}$, 2^{19}, and 2^{21} and distinct disorder strengths $\Delta\eta = 0.5$ and 0.75. We observe that the critical point ($\alpha_c = 2$) is independent of the magnitude of disorder $\Delta\eta$. This trend was also obtained in 1D electronic models with long-range correlated diagonal disorder [13].

4. Summary and conclusions

We studied the propagation of acoustic waves in a one-dimensional medium with scale-free long-range correlated disorder. The random distribution was assumed to have a power spectrum $S(k) \sim 1/k^{\alpha}$. By using a transfer-matrix method we computed the localization length of the allowed acoustic waves. Our results have shown that for $\alpha > 2$ the localization length in the low-frequency region ($\omega < \omega_c$) scales proportionally to the system size, thus suggesting that these acoustic modes are extended. In addition, by using a dynamical method, based on directly solving the scalar wave equation for the propagation of an acoustic wavepacket, we showed that the chain indeed localizes all the frequencies except those in the frequency range below ω_c. Both formalisms provide an accurate estimate of the mobility edge ω_c. In contrast with 1D random media with local correlations, we numerically demonstrated that scale-free correlations promote a localization–delocalization transition in the thermodynamic limit. Within the context of recent studies on acoustic waves in low-dimensional media with correlated disorder [24–27], our numerical results extend some previous statements concerning the existence of an acoustic wave delocalization induced by correlated disorder. Here we showed that a true mobility edge can emerge in systems with strong long-range correlated disorder delimiting a finite range of transmitted frequencies. By following the recent literature on the self-affine long-range correlated disorder distribution, it seems that it plays a universal role in wave propagation phenomena [5, 11, 12, 14]. The critical point ($\alpha_c = 2$) is the same as that obtained in 1D models with long-range correlated on-site energies [5], hopping terms [14], and two-dimensional one-electron Hamiltonians with correlated on-site potentials [11, 12]. We expect that the present work will stimulate further theoretical and experimental investigations along these lines.

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