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Coherent magnon dynamics in ferromagnetic models with nonuniform magnetic field and correlated disorder

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ABSTRACT

In this work we investigated the nature of the one-magnon eigenstates in a disordered chain at the presence of a non-uniform magnetic field. In our study, we analyzed the one-dimensional ferromagnetic Heisenberg model within the one-magnon framework. The spin-spin interaction was considered as a correlated disorder distribution with power law spectrum $S(k) \propto k^{-\alpha}$. By using numerical methods we calculated the time evolution of a initially localized Gaussian wave-packet. Our results reveal that for weak correlations ($\alpha < 1$), the magnetic wave-packet remains localized around the initial position and for $\alpha > 1$, we got an oscillatory profile similar to the Bloch-like phenomenology. We calculate the frequency of these oscillations and observed that it is in a good agreement with the semi-classical approach traditionally used to explain the Block-like oscillatory behavior.

1. Introduction

The localization properties of spin-waves in low dimensional quantum Heisenberg ferromagnets has been target of recent interest. It was demonstrated that the finite energy spin-waves are exponentially localized for any degree of disorder [1]. However, it was also shown that the typical localization length diverges as one approaches to the bottom of the band [1–5]. Moreover, it was proved that the spin-wave dynamics may exhibit a super-diffusive behavior even in the presence of disorder. The effect of correlated disorder on the one-dimensional quantum Heisenberg ferromagnet was investigated by Ref. [6]. By employing mathematical methods such as renormalization group, integration of the equations of motion and exact matrix diagonalization, it was shown that extended states appear for sufficiently strong correlations. Based on Einstein relation, the dynamics of the spin waves in the quantum Heisenberg $S=1/2$ was investigated by analytical means in Refs. [7,8]. The ballistic regime associated with the strong degree of correlations within the disorder distribution was explained from the generalized Langevin equation. Within the context of interacting spin waves in $S=1/2$ ferromagnetic chains, some studies were done in the past (see, for example, Ref. [9]). Because of the spin considered, two spin excitations can never occupy the same site, i.e, this interaction is closely related to an infinite Coulomb repulsion. It was shown that the localization length of high-energy states are rather small but diverges as one approaches the ground state energy [9]. Furthermore, the one-dimensional quantum disordered $S=1/2$ Heisenberg ferromagnetic model with long-range correlated exchange couplings was studied in Ref. [10]. By using a numerical diagonalization of the complete Hamiltonian, the spin-wave participation number was computed. This procedure indicates that, in the regime of strongly correlated random exchange couplings, there are extended spin waves with finite excitation energies. The effect of the kinematic repulsion between the two magnons on the dynamics of two-spin excitations was also investigated. The competition between the kinetic repulsion and ballistic dynamics in the strongly correlated regime leads to a strong degree of spin-spin correlations [10].

Another interesting contribution within the context of spin waves dynamics under effect of interacting terms was made in Ref. [12]. The authors investigated the localization of a single spin-wave state in finite a crystalline Heisenberg model with spin-1/2 under effect of a nonuniform magnetic field. It was demonstrated that as the magnetic field gradient is increased, localization starts at the chain ends and gradually involves magnon states inside the chain [12]. The authors also studied how the superposition of an extended spin wave and a weakly nonuniform magnetic field can promote magnon Bloch-like oscillations (BO). The richness of magnetic systems with interacting terms or nonuniform magnetic field can be observed for example in Refs. [13,14]. In Ref. [13] it was shown that the magnon flow could also be generated by a nonuniform external magnetic field. The transport of magnetization in insulating magnetic systems it was analyzed in Ref. [14]. It was demonstrated that within the ferromagnetic context, the magnetization dynamics can be analyzed as a transmission of magnons

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and the spin conductance depends on the temperature. For antiferromagnetic chains, the spin conductance is quantized and a sort of generalized Hall effect emerges.

In our work we are interested in providing further contribution along these lines. We will adopt a quantum Heisenberg model with spin 1/2 and disordered exchange spin–spin coupling. In our model we will take into account the effect of correlated disorder as well as the presence of a nonuniform magnetic field. We will investigate in details the competition between disorder, correlations and external magnetic field. The spin–spin interaction will be considered as a correlated disorder distribution with power law spectrum $S(k) \propto k^{-\alpha}$. For $\alpha=0$ we recover the standard Heisenberg model with uncorrelated disordered local couplings. For $\alpha > 0$, the model contains intrinsic correlations within the spin–spin couplings. We will solve the Schrödinger equation by using a precise numerical formalism and calculates the spin-wave evolution. Our numerical calculations suggest that, for weak correlations ($\alpha < 1$), the magnetic wave-packet remains localized around the initial position. For $\alpha > 1$ our results reveal an oscillatory profile similar to Bloch-like oscillations. Therefore, our calculations suggests the possibility of one-magnon's Bloch oscillations in chains with correlated disorder. We calculate the frequency of these oscillations and observed that this frequency is in a good agreement with the spatial derivative of the energy interaction between the magnon and the magnetic field. Moreover, a semi-classical approach was also used to give a better explanation of the oscillatory behavior found.

2. Model and formalism

Our model consists of a quantum ferromagnetic Heisenberg model with N spins 1/2 under effect of a nonuniform magnetic field $\vec{H}_n$. According to [6,12] we write the Hamiltonian below:

$$\mathcal{H} = -\sum_{n=1}^{N} \{J_n \vec{S}_n.\vec{S}_{n+1} + \vec{H}_n.\vec{S}_n\} \tag{1}$$

where J_n represent the exchange couplings connecting sites n and $n+1$. In our work we will consider J_n as the trace of a fractional Brownian motion with power spectrum $S(k) \propto k^{-\alpha}$ [6,15–18]. To build this sequence, we generate J_n by using the following procedure: First we generate the sequence V_n defined as:

$$V_n = \sum_{k=1}^{N/2} (k^{-\alpha/2}) \cos(2\pi kn/N + \Phi_k) \tag{2}$$

where k is the wave vector of the modulations on the random coupling landscape and Φ_k are $N/2$ random phases uniformly distributed in the interval $[0, 2\pi]$. The exponent α controls the degree of correlations within the sequence $\{V_n\}$. Then, we normalize the sequence in order to keep the mean value zero and the standard deviation equal to unit: $V_n^* = (V_n - \langle V_n \rangle)/\sqrt{\langle V_n^2 \rangle - \langle V_n \rangle^2}$. After these two steps, in order to assure nonzero spin interaction, we define exchange coupling as: $J_n = V_n^* + 4.5$. The nonuniform magnetic field is defined as $\vec{H}_n = [(\nu_B \eta H_0)n]\vec{z} = (Hn)\vec{z}$ [12]. We are interested in studying the one-magnon subspace of this Hamiltonian. The typical time-dependent wave-function of such excitation is given by $|\phi(t)\rangle = \sum_n f_n|n\rangle$ where $|n\rangle$ represents a wave-function of the chain state with a single reversed spin at site n ($|n\rangle = S_n^-|0\rangle$ where $|0\rangle$ is the ferromagnetic ground state). The time-dependent Schrödinger equation can be written as:

$$S(J_n + J_{n-1})f_n(t) - SJ_n f_{n+1} - SJ_{n-1} f_{n-1} + H_n f_n(t) = i\hbar \frac{df_n(t)}{dt} \tag{3}$$

We emphasize that $H_n = Hn$ is the magnetic field at the n-th spin. We obtained the numerical solution of Eq. (3) for a given initial condition $|\phi(t=0)\rangle$ by following the formalism described in [6]: First we diagonalize the Hamiltonian described in Eq. (1) in order to find all eigenfunctions $\{|\psi_j\rangle = \sum_m c_m^j |m\rangle\}$ and its associated eigenvalues $\{E_j\}$. Using these eigenfunctions we expand $|\phi(t=0)\rangle$ as:

$$|\phi(t=0)\rangle = \sum_j (\langle \phi(t=0)|\psi_j\rangle)|\psi_j\rangle \tag{4}$$

Then, by using that $|\phi(t)\rangle = e^{-i\mathcal{H}t}|\phi(t=0)\rangle$ we can find $f_n(t)$ with the following equation:

$$f_n(t) = \sum_j \{Z_j c_n^j e^{-iE_j t}\} \tag{5}$$

where $Z_j = \sum_m f_m(t=0) c_m^j$. This formalism is stable, fast and keeps the wave-function normalized ($\sum_n |f_n(t)|^2 = 1$) along the entire time interval. We emphasize that we can solve Eq. (3) by using standard methods like Runge–Kutta, for example. However, due to the presence of the diagonal energy associated with the magnetic field, it is much more difficult to perform the numerical integration and keep the wave-function normalized. In our work we calculate the magnon position $\langle x(t)\rangle$ defined as:

$$\langle x(t)\rangle = \sum_n n|f_n(t)|^2 \tag{6}$$

3. Results and discussions

In our calculations, the initial wave-packet was set with a Gaussian distribution localized at the center of chain. Therefore, the initial condition was defined as $|\phi(t=0)\rangle = \sum_n f_n(t=0)|n\rangle$ where $f_n(t=0) = Ae^{-(n-N/2)^2/4}$ and A is a normalization constant. Calculations were made for $N=500$ spins. *We are using open boundary conditions*. In Fig. 1 we plot the magnon position $\langle x(t)\rangle$ versus time for $\alpha = 0, 0.5, 1$ and $H=0$ up to 2. We observe that the magnon remains localized around the initial position and exhibits an incoherent oscillatory behavior. We emphasize that for $\alpha=1$ and for strong values of H ($H=1$ and 2) the magnon's position exhibits a quasi-coherent dynamics thus suggesting a oscillatory framework. In fact, as the degree of correlations is increased ($\alpha > 1$) we verified a magnon's oscillation around the initial position. We see these results in Fig. 2 for $\alpha = 1.5, 2, 2.5$. The magnon remains localized around the initial position however, $\langle x(t)\rangle$ exhibits an oscillatory behavior quite compatible with the well known Bloch oscillation phenomenon. We also noticed that as the H value is increased, the size of region in which that the magnon oscillates decreases. So, we find another similarity with electronic Bloch oscillations.

In Fig. 3 we plot $|f_n|^2$ versus n and t for $\alpha=0$ and 2.5 and $H=0.5$. An oscillatory profile is observed for $\alpha = 2.5$. For $\alpha=0$, the 3d plot of the wave-packet is in a good agreement with results shown in Fig. 1: the magnon remains localized and coherent oscillations are not evident. Our results in fact suggests that this ferromagnetic model under effect of nonuniform magnetic field and strong correlated disorder exhibits Bloch-like oscillations. We stressed that for large α, the one-magnon disordered Hamiltonian shows a phase of extended states at the low-energy region [6]. In this way, our results suggest that the one-magnon wave-packet oscillates in that band. We will return to discuss this point in details at the end of this section. The most important properties of the Bloch Oscillations phenomenology is its intrinsic frequency and the size of region where the particle oscillates. We analyzed the frequency of these oscillation by computing the Fourier transform of $\langle x(t)\rangle$ for distinct values of H. In Fig. 4 we plot the Fourier transform of $\langle x(t)\rangle$ $(x(\omega))$ computed for $\alpha=2.5$ and $H = 0.5, 1, 2$. It seems that

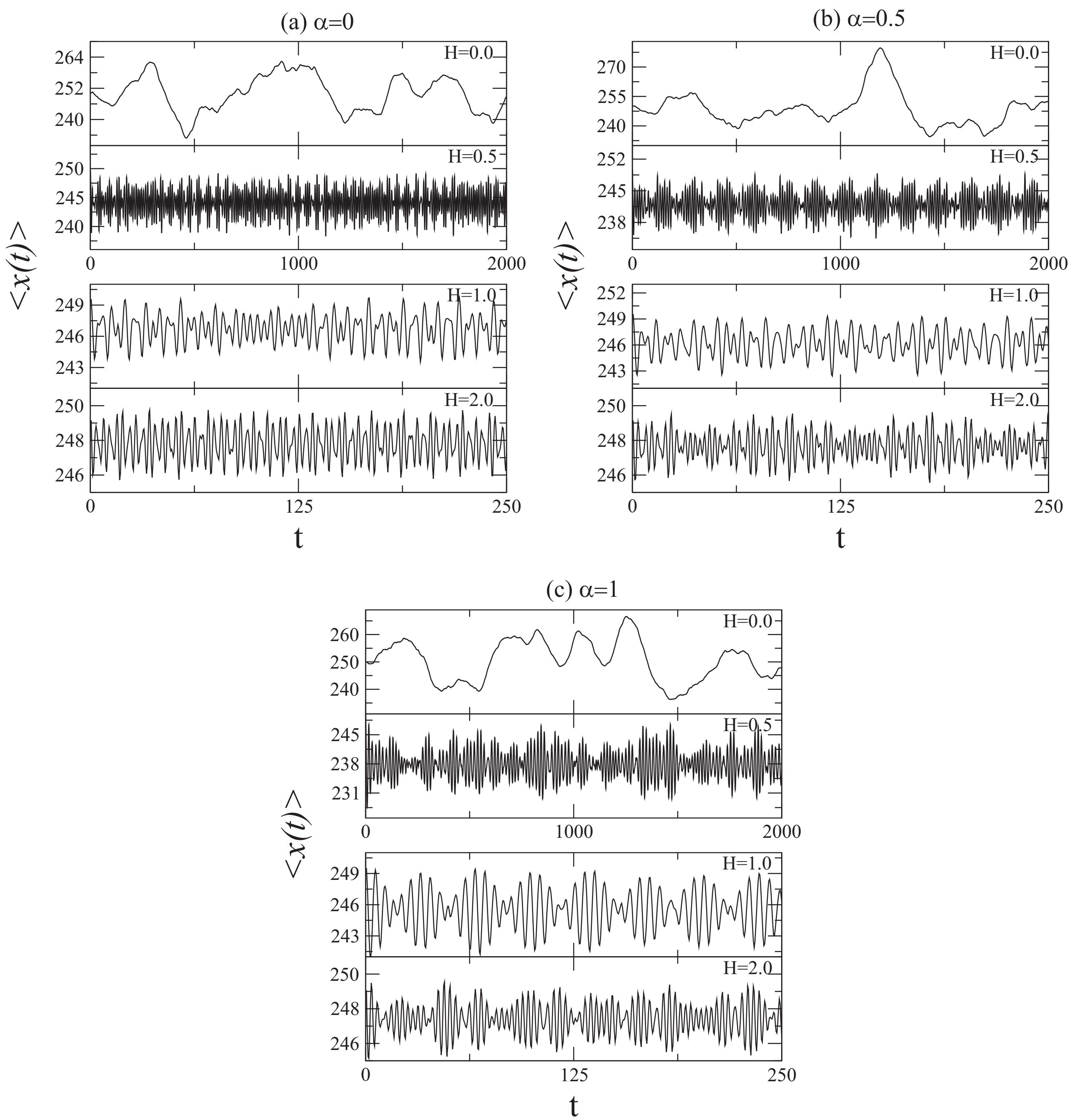

Fig. 1. The magnon position $\langle x(t)\rangle$ versus time for $\alpha = 0$, 0.5, 1 and $H=0$ up to 2. We observe that magnon remains localized around the initial position. For $\alpha=1$ and for strong values of H ($H=1$ and $H=2$) the magnon's position exhibits a quasi-coherent oscillatory framework.

the magnon's oscillatory behavior has a dominant frequency $\omega \approx H$. This is an interesting result which strengthens the previous statement about the Bloch-like oscillation of the one-magnon wave-packet.

Following the semi-classical theory for electronic Bloch-oscillation [18], it is well known that in disorder-free systems, a uniform electric field F causes the dynamic localization of the electron and gives rise to an oscillatory motion of the wave packet (Bloch's oscillation). The period of these oscillations can be estimated as $\tau_B = 2\pi/F$, i.e. the frequency is $\omega = F$. By comparing the energy interaction of an one-electron with an uniform electric field F ($E_e = Fn$) to the diagonal term in our Schrödinger equation (see Eq. (3)) we realize that H is analogous to F within the one-magnon theory. Similarities between the static electric field F within the electronic model and the spatial derivative of the magnetic energy H were also anticipated in Ref. [12].

Then, by using these simple arguments we understand the frequency of oscillations $\omega \approx H$ obtained in Fig. 4. We emphasize that the analogy between the static electric field F within the electronic model and the spatial derivative of the magnetic energy H was also anticipated in Ref. [12].

Another key signature of the Bloch's oscillation phenomenon is noticed by direct relation between the size L of the segment over which the particle oscillates and the width W_b of free particle's band. By adapting the semi-classical theory to the one-magnon formalism, the relation should be: $L = W_b/H$. Estimates of the width of the extended one-magnon states (W_b) can be done by considering the Heisenberg Hamiltonian in the absence of

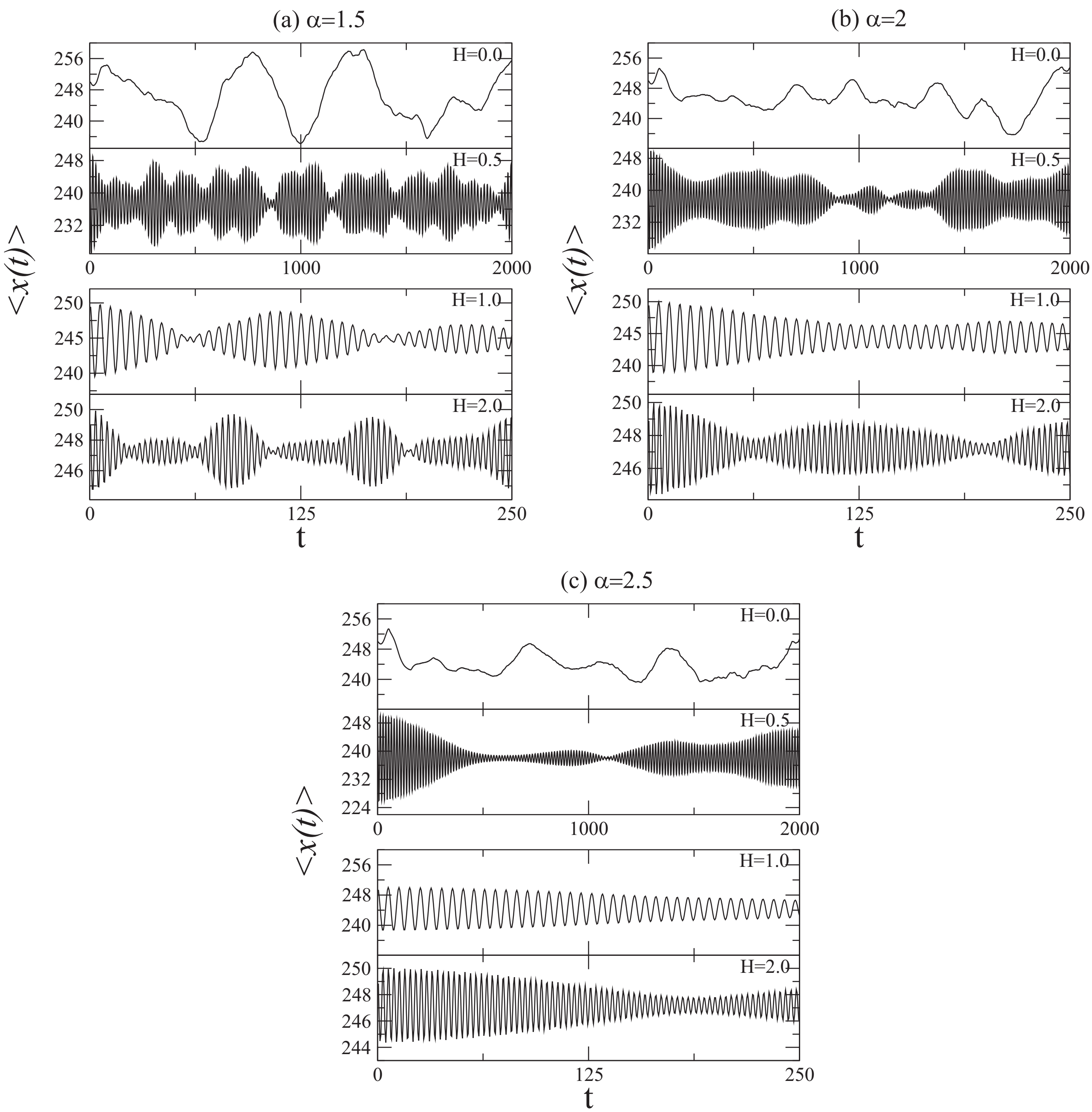

Fig. 2. For $\alpha > 1$ the magnon remains localized around the initial position and exhibits a Bloch-like oscillatory profile.

magnetic field ($H=0$) and by applying a transfer matrix formalism [19,20] in order to calculate the inverse of the localization length $1/\lambda = \{\lim_{N\to\infty}(1/N)\log[|Q_N F(0)|/|F(0)|]\}$. Here, $F(0) = \binom{f_1}{f_0}$ is a generic initial condition and Q_N is the product of all transfer matrices. The inverse of localization length is used to understand the nature of one-magnon eigenstates and to estimate the width of band of extended magnons. Extended states exhibit $1/\lambda \to 0$ while localized ones displays $1/\lambda$ finite. In Fig. 5 we plot the inverse of the localization length $1/\lambda$ versus energy for $\alpha = 1.5, 2, 2.5$. As a result, $1/\lambda$ vanishes in a finite low energy region. We also observe that the size of this region increases as the degree of correlations is increased. Within our model, the size of this energy window represent the width of the band of extended one-magnon states (i.e. W_b). Therefore, by estimating W_b from Fig. 5 we may compare it to the amplitude L of the oscillation. Following Fig. 5 $W_b \approx 4.5$ for $\alpha = 1.5$, $W_b \approx 5.$ for $\alpha=2$ and $W_b \approx 6.$ for $\alpha=2.5$. By those values, we verify that the amplitude L of the magnon oscillation roughly agrees with the semi-classical prediction $L \approx W_b/H$.

4. Summary and conclusions

We have studied the spin 1/2 one-dimensional quantum ferromagnetic Heisenberg model with disordered spin–spin exchange couplings and a non-uniform magnetic field. In our study, we analyzed the one-dimensional ferromagnetic Heisenberg model within the one-magnon framework. The spin–spin interaction was considered as a long-range correlated disorder distribution with power law spectrum $S(k) \propto k^{-\alpha}$. We have also introduced a non-uniform magnetic field according to the form $\vec{H}_n = Hn\vec{z}$. By using numerical methods we calculated the time evolution of an initially localized Gaussian wave-packet. Our

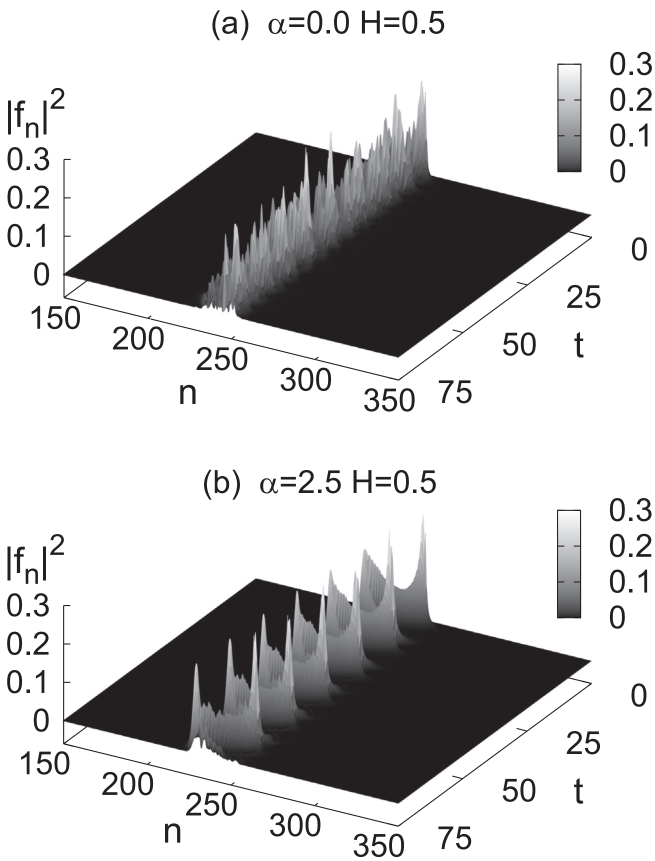

Fig. 3. $|f_n|^2$ versus n and t for $\alpha=0$, 2.5 and $H=0.5$. We observe that the oscillatory profile appears clearly for $\alpha=2.5$ while for $\alpha=0$ the magnon remains trapped around the initial position.

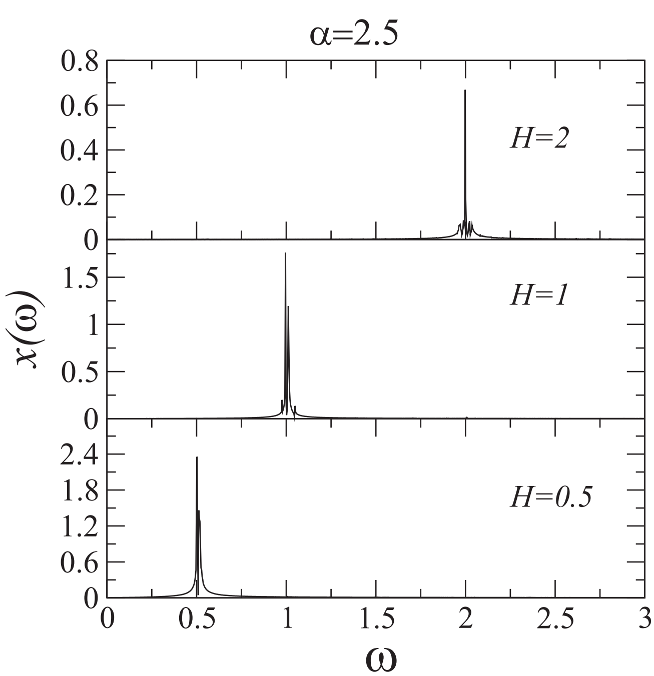

Fig. 4. Fourier transform of the magnon position $x(\omega)$ versus ω computed for $\alpha=2.5$ and $H = 0.5$, 1, 2. Our calculations suggests that the magnon's oscillatory behavior exhibits a dominant frequency $\omega \approx H$.

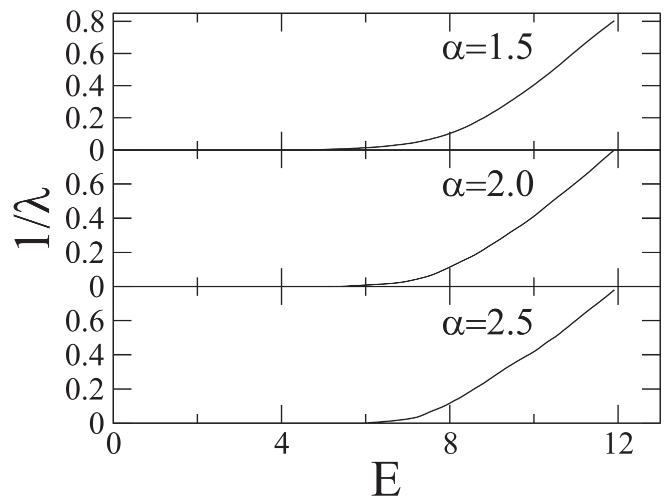

Fig. 5. The inverse of localization length s $1/\lambda$ versus energy computed for $H=0$ and $\alpha = 1.5$, 2, 2.5. Extended states exhibits $1/\lambda \to 0$ and localized modes displays $1/\lambda$ finite.

numerical calculations indicate that for weak correlations $(\alpha < 1)$ the magnetic wave-packet remains localized around the initial position. For $\alpha > 1$ we have obtained an oscillatory behavior similar to Bloch-like oscillations. We investigate the frequency and the amplitude of these oscillations and, in our numerical tolerance, the results were quite compatible with the semi-classical treatment of that phenomenon. Therefore, our results indeed suggest the possibility of one-magnon's Bloch oscillations in ferromagnetic chains with correlated disorder. We emphasize that this is an interesting and new result once, in general, Bloch-like oscillations are found in crystalline systems [12]. In our work, we demonstrated by numerical means that, even at the presence of strong disorder, it is possible to stabilize coherent spin-wave oscillation with controlled frequency. We hope that the present work will stimulate further studies on the transport of magnetic excitations in correlated disordered magnets.

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