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Unveiling Acoustic Modes in Two-Dimensional Systems with Exponential Correlations in Disorder

M.O. Sales[1], L. D. da Silva[2], M. S. S Junior[3], and F. A. B. F. de Moura[3]

1) IFMA Campus São João dos Patos, rua Padre Santiago, s/n, Centro, São João dos Patos-MA, 65665-000, Brazil.

2) Escola Estadual Professor Theotônio Vilela Brandão, R. Cel. Adauto Gomes Barbosa, s/n - Poço, Maceió - AL, 57025-045

3) Instituto de Física, Universidade Federal de Alagoas, Maceió, AL, 57072-970, Brazil

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In this study, we will analyze how acoustic modes propagate within a rectangular system exhibiting disorder in the compressibility term. Exponential correlations characterize the distribution of disorder. Our main objective is to investigate the behavior and velocity of harmonic mode packets as they traverse through this system. To achieve this, we will use a high-order finite difference formalism. We will also examine how the propagation is affected by the spectral structure of the incident pulse. We aim to understand better the interdependence between the system's correlations and the modes' behavior.

Keywords: tight-binding model, correlated disorder, localization

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1. Introduction

In wave propagation, acoustic systems serve as quintessential models for understanding the intricate interplay between waves and the medium through which they travel.[1–7] While the behavior of sound waves in ordered, homogeneous media has been extensively studied and understood, the dynamics in disordered systems present a rich tapestry of complexity yet to be fully unraveled.[8] In recent years, exploring acoustic modes in disordered systems has emerged as a focal point, captivating researchers across disciplines ranging from physics to engineering and materials science.Disordered systems, characterized by irregularities or randomness in their structure, pose unique challenges and opportunities for studying wave phenomena. Unlike their ordered counterparts, where wave propagation follows predictable paths and behaviors, disordered systems exhibit many intriguing phenomena such as multiple scattering, Anderson localization, and mode hybridization.[9,10] These phenomena arise from the complex interplay between wave interference and disorder-induced scattering, giving rise to rich and often unexpected wave dynamics.[11] Understanding the behavior of acoustic modes in disordered systems holds significant implications across various fields. In materials science, for instance, the ability to manipulate and control acoustic waves in disordered media opens new avenues for designing novel materials with tailored acoustic properties, such as enhanced sound insulation or wave guiding.[12] In photonics and optoacoustics, disordered systems offer platforms for developing robust and efficient light-matter interactions and devices, promising advancements in areas like random lasers and optical communication.[13] Moreover, studying acoustic modes in disordered systems extends beyond fundamental research, with practical applications ranging from non-destructive testing and medical imaging to seismic wave analysis and telecommunications.[14,15] Researchers aim to harness the inherent complexity of innovative technological solutions and practical advancements by elucidating the intricate mechanisms governing wave propagation in disordered media.

The problem of elastic waves in heterogeneous media characterized by off-diagonal disorder and long-range correlations was investigated in ref.[16] The authors explore how these types of disorders and correlations affect the behavior of elastic waves within the medium. In reference,[17] researchers delved into the phenomenon of acoustic wave localization within one-dimensional models with chaotic elasticity. The authors scrutinized the behavior of localized modes within these chaotic systems through numerical calculations. Meanwhile, in ,[18] authors explored the propagation of acoustic waves in two-dimensional disordered media exhibiting specific types of short- and long-range correlations. Our article embarked on a captivating exploration of acoustic modes within disordered systems. This study investigates the propagation of acoustic modes in a rectangular system afflicted by disorder in the compressibility term. We will assume that the disorder distribution incorporates exponential correlations in its composition. Our analysis will examine the propagation of harmonic mode packets within this system. By employing a high-order finite difference formalism, our investigation aims to discern the impact of these correlations on mode propagation and velocity. Furthermore, we will explore how this propagation correlates with the spectral structure of the incident pulse.

2. Model and Numerical calculation

The propagation of acoustic waves in two-dimensional $N \times M$ disordered systems with constant density at point $\vec{r}$ can be studied by solving the scalar wave equation.[18]

$$\frac{\partial^2 \phi(\vec{r},t)}{\partial t^2} = \vec{\nabla} \cdot [\eta(\vec{r})\vec{\nabla}\phi(\vec{r},t)], \tag{1}$$

where $\eta(\vec{r})$ represents the bulk compressibility of the medium at point $\vec{r}$. The right side of the previous equation can be expressed as:

$$\begin{aligned}\vec{\nabla} \cdot [\eta(\vec{r})\vec{\nabla}\phi(\vec{r},t)] &= \partial_x \eta(\vec{r}) \partial_x \phi(\vec{r},t) + \partial_y \eta(\vec{r}) \partial_y \phi(\vec{r},t) \\ &\quad + \eta(\vec{r})[\partial_x^2 \phi(\vec{r},t) + \partial_x^2 \phi(\vec{r},t)]\end{aligned} \tag{2}$$

The acoustic wave equation in two dimensions can be discretized using finite differences. Let's define a rectangular mesh where the acoustic mode $\phi(\vec{r},t)$ can be represented as $\phi^n_{i,j}$, where $x = i\delta x$, $y = j\delta y$, and $t = n\delta t$. The bulk compressibility $\eta(\vec{r})$ will be also represented as $\eta_{i,j}$. We stres that $i = 1,2,3,...,N$, $j = 1,2,3,...,M$, $\delta x = \delta y = 1$ and $\delta t \approx 10^{-3}$. Utilizing fourth-order spatial and second-order temporal finite differences, the finite-difference equations can be written as:[18–21]

$$\frac{\partial^2\phi(\vec{r},t)}{\partial t^2} \approx \frac{\phi^{n+1}_{i,j} - 2\phi^n_{i,j} + \phi^{n-1}_{i,j}}{\delta t^2} \tag{3}$$

,

$$\partial_x^2\phi(\vec{r},t) \approx \frac{-\phi^n_{i+2,j} + 16\phi^n_{i+1,j} - 30\phi^n_{i,j} + 16\phi^n_{i-1,j} - \phi^n_{i-2,j}}{12(\delta x)^2} \tag{4}$$

and

$$\partial_x\phi(\vec{r},t) \approx \frac{-\phi^n_{i+2,j} + 8\phi^n_{i+1,j} - 8\phi^n_{i-1,j} + \phi^n_{i-2,j}}{12\delta x} \tag{5}$$

We emphasize that we will use similar expressions for the partial derivatives with respect to the y-direction:

$$\partial_y^2\phi(\vec{r},t) \approx \frac{-\phi^n_{i,j+2} + 16\phi^n_{i,j+1} - 30\phi^n_{i,j} + 16\phi^n_{i,j-1} - \phi^n_{i,j-2}}{12(\delta y)^2} \tag{6}$$

and

$$\partial_y\phi(\vec{r},t) \approx \frac{-\phi^n_{i,j+2} + 8\phi^n_{i,j+1} - 8\phi^n_{i,j-1} + \phi^n_{i,j-2}}{12\delta y} \tag{7}$$

The partial derivatives of the bulk compressibility $\eta(\vec{r})$ can be written as:

$$\partial_x\eta(\vec{r}) \approx \frac{-\eta_{i+2,j} + 8\eta_{i+1,j} - 8\eta_{i-1,j} + \eta_{i-2,j}}{12\delta x} \tag{8}$$

and

$$\partial_y\eta(\vec{r}) \approx \frac{-\eta_{i,j+2} + 8\eta_{i,j+1} - 8\eta_{i,j-1} + \eta_{i,j-2}}{12\delta y} \tag{9}$$

By combining equations 3 through 9 with equation 1, we can construct a recursive finite-difference equation to obtain the acoustic wave $\phi(\vec{r},t)$. In our model, the bulk compressibility $\eta_{i,j}$ will be distributed following a disorder distribution with intrinsic exponential correlations. Initially, we will calculate the 2D distribution defined as:

$$a_{i,j} = \sum_{o,p} e^{-(\sqrt{(i-o)^2+(j-p)^2}/L_0)}[\rho_{o,p}] \tag{10}$$

where $\rho_{o,p}$ are $N \times M$ random numbers within the interval $[-1,1]$ and L_0 represents an effective intensity of the correlation degree inside the distribution. The summation over o,p indicated in the previous formula can be efficiently performed considering that, for a given pair i,j, only nearby terms are relevant; the exponential decay eliminates terms for which $|i-o|$ and $|j-p| > 150$. Thus, for each pair (i,j), we will sum the values of (o,p) while ensuring that $|i-o| < 150$ and $|j-p| < 150$. For values of $L_0 \leq 100$, performing the summation using this technique does not alter the final result in any significant manner. We emphasize that this formula is not defined for $L_0 = 0$. We normalize the 2D distribution $a_{i,j}$ such that $\langle a_{i,j} \rangle = 0$ and $\langle a_{i,j}^2 \rangle = 1$. *The bulk compressibility is then defined as $\eta_{i,j} = \tanh(a_{i,j})+2$. This transformation ensures that compressibility values remain positive and bounded, effectively avoiding unphysical values such as zero or negative compressibility. Importantly, although the hyperbolic tangent introduces nonlinearity, it is a smooth and monotonic function that preserves the spatial structure of correlations originally present in $a_{i,j}$. Therefore, the exponential decay of correlations in the original disorder is maintained in the transformed field $\eta_{i,j}$, ensuring that the essential features of the correlated disorder are not lost. We would like to emphasize that exponentially correlated disorder is not only a theoretically convenient assumption, but also a physically relevant feature in a wide range of real-world systems.[22, 23] In many disordered materials, spatial fluctuations in physical properties such as stiffness, mass density, or refractive index exhibit correlations over finite distances, resulting in an exponential decay of spatial correlations. For instance, in porous media and composite materials, structural heterogeneity is frequently modeled using exponentially correlated noise to reflect the finite correlation length of material parameters.[24] Similar behavior has been reported in biological tissues and soft matter systems,[25] as well as in models of random elastic media.[26, 27] In seismology and geophysics, exponentially correlated disorder is also employed to describe wave propagation in heterogeneous subsurface structures.[28] These examples highlight the physical significance and applicability of exponentially correlated disorder when modeling acoustic or vibrational transport in complex media.*

We will evaluate the statistical properties of the distribution $\eta_{i,j}$ by, for instance, numerically computing the autocorrelation function defined as $C(R = |\vec{R}|) = [< \eta(\vec{r})\eta(\vec{r}+\vec{R}) > - < \eta(\vec{r}) >< \eta(\vec{r}+\vec{R}) >]/[< \eta(\vec{r})^2 > - < \eta(\vec{r}) >^2]$. In addition to autocorrelation, we will also calculate the probability distribution $P(\eta_{i,j})$ and the average local disorder Δ. Local disorder is calculated as follows: we consider a sample of size $N \times M$. We divide this sample into k boxes of size $d_0 \times d_0$ with $d_0 = 50$. We calculate the local disorder in each of these boxes as $\sigma_k = \sqrt{< \eta_{i,j}^2 >_k - < \eta_{i,j} >_k^2}$ where $<>_k$ denotes an average within box k. The average local disorder is defined as: $\Delta = \sum_k \sigma_k / N_k$ where N_k is the number of boxes within the sample. In our calculations, we will use $N \times M = 300 \times 3000$. In Figure 1, we present a brief summary of the main results of this analysis. Figure 1(a) displays the autocorrelation function $C(R) \times R$ for several values of the correlation length parameter $L_0 = 1, 10, 20, 30$. For $L_0 = 1$, the autocorrelation decays very rapidly: even for small distances $R > 0$, the values of $C(R)$ drop close to zero, indicating that the disorder is essentially uncorrelated beyond immediate neighbors. In contrast, for larger values of L_0, the decay of $C(R)$ becomes slower, and the function maintains significantly positive values over a broader range of distances. This behavior reflects the increased spatial coherence in the disorder: as L_0 increases, the region over which the compressibility remains positively correlated becomes larger. In other words, the spatial extent of the correlated disorder grows with L_0, as clearly illustrated by the widening of the region where $C(R) > 0$. In Figure 1(b), we further visualize the local disorder of the compressibility field, emphasizing how increasing L_0 leads to smoother spatial fluctuations and more extended correlated regions. It can be observed that Δ decreases as L_0 increases. However, we notice a plateau in the region $L_0 > 50$, indicating that this type of correlated disorder still maintains effective local disorder. Finally, in Figure 1(c), we present the probability distribution $P(\eta_{i,j}) \times \eta_{i,j}$. Our results suggest that, at least within the considered range, correlation does not drastically alter the profile of the probability distribution. We have considered $L_0 \leq 100$ in our calculations, and no significant changes were found in $P(\eta_{i,j})$. Within our study, it makes little sense to consider $L_0 > 100$ as we would have a system in which the correlation length would be close to the sample dimensions.

3. Results

Our primary analysis of the eq. 1 involves a numerical experiment to directly measure a pulse's propagation throughout the system. One side of our rectangular lattice is coupled with oscillators that inject a pulse defined as $\phi_{i,j=0}(t) = \sum_{\omega_n} \Pi \cos(\omega_n t)$, where Π represents a small amplitude ($\Pi = 0.001$), and ω_n is a set of frequencies within the interval $[0.01, 5]$. To analyze the propagation throughout the system, we monitored the time evolution of the pulse by tracking the wave at position $[N/2, M/2]$. Subsequently, we computed $Z(\omega) = |F(\phi_{N/2,M/2}(t))|$,[19] where $F(A)$ represents the Fourier transform of function A. The quantity $Z(\omega)$ provides insights into the frequencies propagating along the sample. If $Z(\omega) \approx 0$, the frequency ω does not propagate along the lattice. Conversely, if $Z(\omega) > 0$, our results demonstrate numerically that acoustic modes with frequency ω evolve along the lattice from one side to the other. We have used $\delta t = 10^{-3}$ in our calculations. However, we have also conducted some experiments with $\delta t = 10^{-4}$ or 10^{-5} and the results were the same. It is important to note that we do not apply our method directly to the entire sample of size $N \times M$. Instead, we will start with a size $N \times M_0$ system, where $M_0 = 100$. As the wave reaches the right edge of the system, we will increase the value of M_0, which will be limited to a maximum value of M. The final time we used in our study was around 2000 time units, during which the wave at the final right edge was almost negligible. In fig. 2 we plot $Z(\omega) \times \omega$ considering $L_0 = 1, 10, 20, 30$. We emphasize that we have monitored the acoustic wave at position $[N/2, M/2]$; however, we start collecting data about $\phi_{N/2,M/2}(t)$ after the wave arrives at this point, that is after the $|\phi_{N/2,M/2}(t)|$ becomes more significant than 10^{-5}. After this moment, we collected approximately 400 time units and then performed the Fourier transform to calculate Z. We also utilize approximately 20 distinct samples to calculate averages and enhance the quality of curves. We can observe in Figure 2 that the function Z for $L_0 = 1$ becomes nonzero only in the

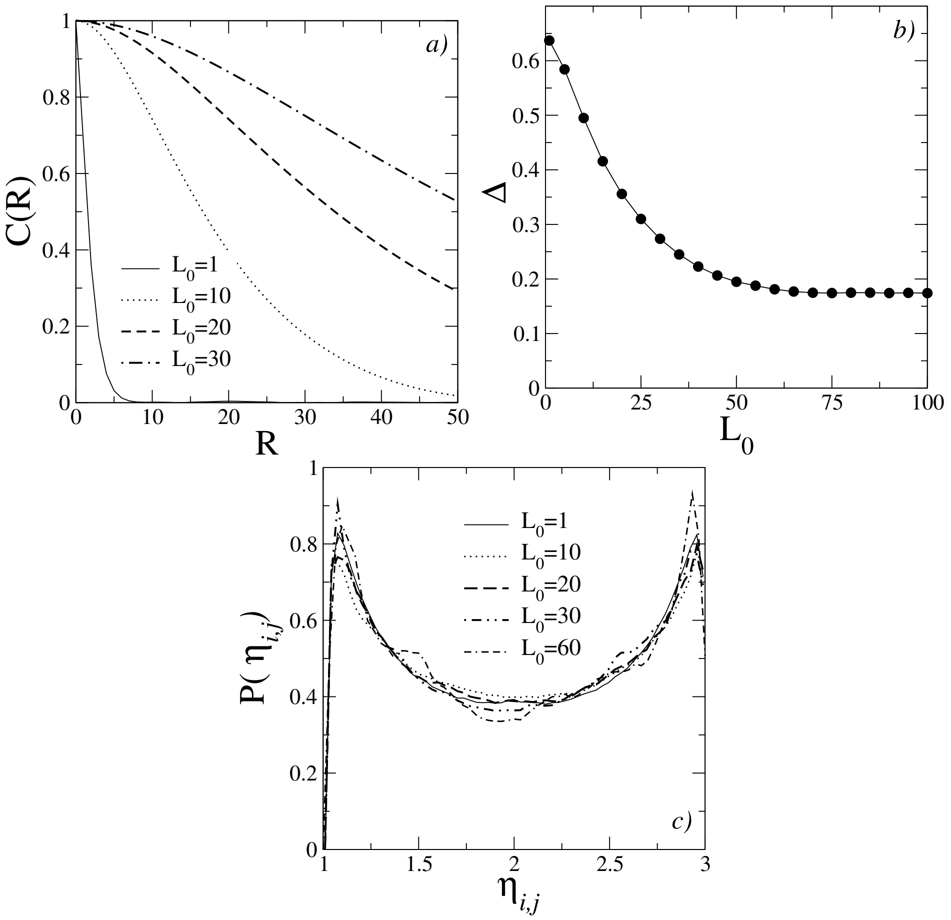

Fig. 1. *(a) Autocorrelation function $C(R) \times R$ for different correlation lengths L_0. For $L_0 = 1$, correlations decay rapidly, while larger L_0 values lead to broader regions where $C(R)$ remains significant, consistent with an exponential decay $C(R) \sim \exp(-R/L_0)$. (b) Average local disorder amplitude as a function of L_0, showing that spatial variability decreases as the correlation length increases. (c) Probability distribution of the local compressibility $\eta_{i,j} = \tanh(a_{i,j}) + 2$, where $a_{i,j}$ is a normalized random field with exponential correlations. The transformation ensures that $\eta_{i,j}$ remains positive and bounded, while preserving the original spatial correlations present in $a_{i,j}$.*

low-frequency region ($\omega < 0.5$). This result is possibly a consequence of the Anderson localization phenomenon. Low-frequency modes have long wavelengths and, thus, are less sensitive to disorders present in the system. On the other hand, for $L_0 > 1$, a range of frequencies exists approximately in the region $[0, 2]$ where Z is nonzero. This result suggests that harmonic modes with frequencies in this range, initially pumped along the sample, can propagate along the lattice. The function Z becomes nearly zero again for $\omega >> 2$ even for large L_0. This result is similar to previous findings in one-dimensional acoustic systems with correlated disorder.

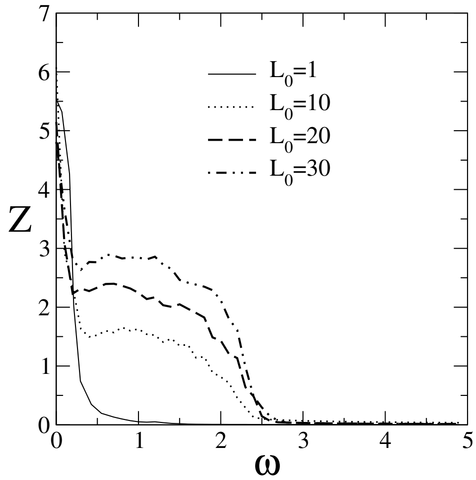

Fig. 2. Numerical calculations of $Z(\omega) \times \omega$. We stress that $Z(\omega)$ is the modulus of the Fourier transform of $\phi_{N/2,M/2}(t)$.

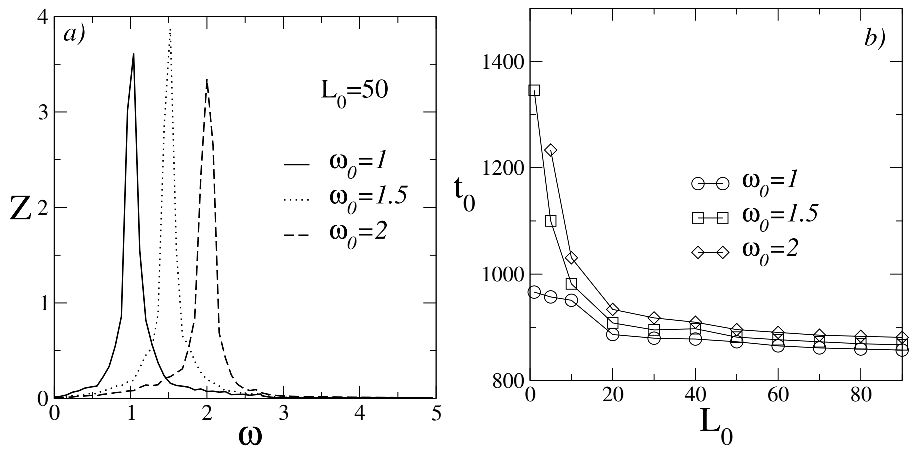

Fig. 3. a) $Z(\omega) \times \omega$ for incident pulses with dominant frequencies $\omega_0 = 1, 1.5, 2$. We have considered a disordered sample with size 300×3000 and correlation degree $L_0 = 50$. b) We estimate the time t_0 that the pulses take to reach the position $[150, 1500]$. Our calculations indicate that high-frequency modes are slightly slower. The velocity also increases as L_0 is increased.

Generally, in 1D systems, acoustic modes become more propagative when correlations are present in the disorder distribution.[17] Here, we demonstrate that acoustic modes, even those of high frequency, can propagate more freely in two-dimensional systems with correlated disorder.

We will now investigate the dispersive properties of this model. We will change the pumping term on the left-hand side of the lattice to a superposition of harmonic

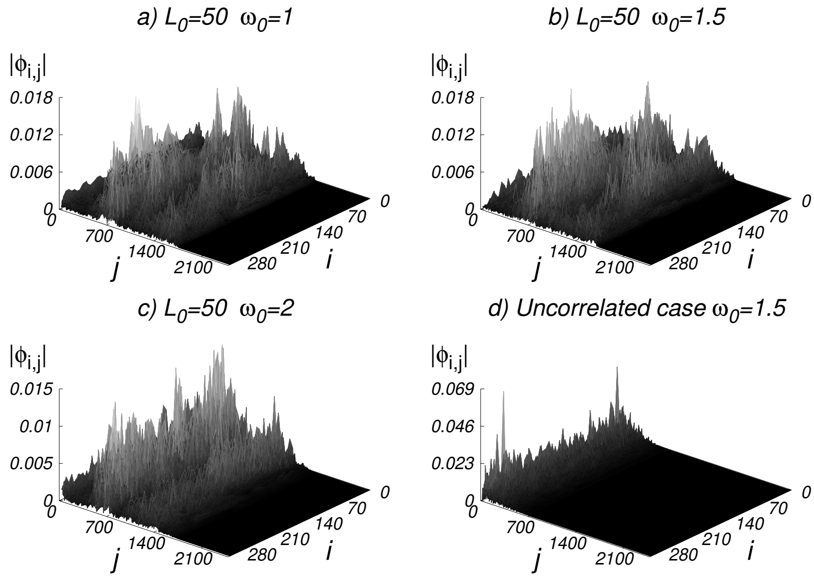

Fig. 4. (a-c) The modulus of the acoustic wave $|\phi_{i,j}|$ versus i and j for $L_0 = 50$. The frequencies considered are: a) $\omega_0 = 1$, b) $\omega_0 = 1.5$, and c) $\omega_0 = 2$. Additionally, we perform the same calculations as in (a-c) but with a dominant frequency of $\omega_0 = 1.5$ and consider a sample with uncorrelated disorder.

modes with frequencies around a given value ω_0, meaning the pumping pulse will be provided by $\phi_{i,j=0}(t) = \sum_{|\omega_n - \omega_0| < 0.05} \Pi \cos(\omega_n t)$. In this manner, we will explore the propagation of a narrow frequency pulse along this model. Let's calculate the Z function for this new pumping condition. Figure 3(a) shows the results of $Z \times \omega$ for $L_0 = 50$ and $\omega_0 = 1, 1.5, 2$. The results clearly show that all three modes can propagate along the system (in good agreements with results found in fig.2). This type of experiment allows investigation into the propagation time scales for each mode. For each value of L_0 and ω_0 under consideration, we will collect the time t_0 for each mode to reach the observation position. The results can be found in Figure 3(b). The findings indicate that lower frequency modes are generally faster. Another interesting observation is that the propagation speed increases slightly as the length location increases; however, it appears to saturate for values of $L_0 > 20$. There is a slight decrease in the region with $L_0 > 20$, but it is insignificant. Moreover, we can see that, for $L_0 < 5$ and $\omega_0 = 2$, the acoustic wave does not reach about the middle of the sample, thus indicating robust localization at this limit. We would like to provide an illustration of the acoustic wave profile for a long time before concluding our study. In figures 4(a-c), we display the modulus of the acoustic wave $|\phi_{i,j}|$ versus i and j for $L_0 = 50$ and frequencies of $\omega_0 = 1$, 1.5, and 2, respectively. In figure 4(d), we present the propagation of a high-frequency mode with a dominant frequency of $\omega_0 = 1.5$ in a sample with uncorrelated disorder. To conduct the experiment shown in case (d), we generate a sequence of uncorrelated disordered numbers $g_{i,j}$ with a Gaussian distribution having a mean of 0 and variance of 1. We calculate the compressibility as $\eta_{i,j} = \tanh(g_{i,j}) + 2$. These calculations of fig. 4 were performed for the final time approximately at $t_{max} \approx 800$. We can see that in the uncorrelated case, the mode becomes localized at the initial part of the sample, with no propagation along the j direction. This is a direct consequence of Anderson's localization in disordered systems. In the case with correlation $L_0 = 50$ (figs. 4(a-c)), we can observe that the acoustic pulse spreads out along the lattice.

4. Summary

Our study investigates the propagation characteristics of acoustic pulses in two-dimensional systems with exponentially correlated disorder, as governed by equation 1. To probe wave propagation, we performed numerical simulations on a rectangular lattice with edge-driven boundary conditions. Pulses were injected along the boundary through oscillators following the expression $\phi_{i,j=0}(t) = \sum_{\omega_n} \Pi \cos(\omega_n t)$, *where* $\Pi = 0.001$ *represents a small amplitude, and the excitation frequencies* ω_n *span the range* $[0.01, 5]$. *The dynamical response at the center of the system,* $[N/2, M/2]$, *was recorded to construct the spectral response function* $Z(\omega)$, *which quantifies the amplitude of transmitted waves at each frequency. The spectral function* $Z(\omega)$ *serves as a diagnostic tool: values of* $Z(\omega) \approx 0$ *indicate frequency components that are effectively blocked (non-propagating), while* $Z(\omega) > 0$ *corresponds to propagating acoustic modes. As shown in Figure 2, for uncorrelated disorder (*$L_0 = 1$*), the spectrum is sharply suppressed for* $\omega \gtrsim 0.5$, *consistent with strong localization effects reminiscent of Anderson localization. In contrast, as the correlation length* L_0 *increases (*$L_0 = 10, 20, 30$*), the range of frequencies capable of propagation broadens substantially, reaching up to* $\omega \approx 2$. *This enhancement in transport with increasing correlation length is consistent with previous findings in 1D disordered systems,*[5, 21] *and it underscores the fundamental role of spatial correlations in shaping wave dynamics. Exponentially correlated disorder, in particular, is not only analytically tractable but also physically realistic in a variety of complex media, including porous materials, soft matter, biological tissues, and composite materials.*[22, 24, 28] *These correlations introduce a finite memory scale into the system, which can effectively suppress localization and restore partial wave coherence over intermediate distances.*

We further analyzed the system's dispersive behavior by tuning the excitation to harmonic modes centered around specific frequencies ω_0 *and monitoring the propagation timescale across the lattice. Our simulations reveal that low-frequency modes tend to propagate faster and more efficiently. Additionally, the propagation speed exhibits a mild increase with* L_0, *saturating for* $L_0 \gtrsim 20$. *For short correlation lengths (*$L_0 < 5$*) and higher excitation frequencies (*$\omega_0 = 2$*), the system enters a localized regime, where waves are unable to reach the center of the lattice, further confirming the suppressive effect of disorder in the absence of long-range correlations.*

Overall, our results highlight the importance of disorder correlations in controlling acoustic transport. The exponential correlation model, due to its physical relevance and tunability, provides a powerful framework for understanding the transition between localized and extended vibrational states. Our findings are in qualitative agreement with previous studies of vibrational and acoustic transport in disordered systems,[23, 29, 30] *and they suggest new avenues for controlling wave propagation through engineered disorder. We hope that the methodology and results presented here inspire further research into wave dynamics in complex media with structured randomness.*

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