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Resonant states and wavepacket super-diffusion in intra-chain correlated ladders with diluted disorder

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Abstract

In this work, we study a tight-binding Hamiltonian model system of a binary correlated ladder with diluted disorder. We introduce intra-chain correlations between the on-site potentials by imposing that $\epsilon_{i,s} = -\epsilon_{i,-s}$ where $s = \pm 1$ indexes the two ladder chains. Further, we consider each ladder chain as composed of inter-penetrating ordered and random sub-chains. We show that the presence of a random on-site distribution in one of the inter-penetrating chains leads to Anderson localization except at a specific symmetric pair of energy eigenmodes. Further, by integrating the time-dependent Schroedinger equation, we follow the time-evolution of an initially localized one-electron wavepacket. We report that the remaining delocalized resonant modes are responsible for a super-diffusive spread of the wavepacket dispersion while the wavepacket participation function remains finite. A scaling analysis of the wavepacket distribution shows that it obeys a universal scaling form with the development of a power-law tail followed by a super-diffusively evolving cutoff. We obtain three exponents characterizing this super-diffusive dynamics and show that they satisfy a simple scaling relation.

1. Introduction

The scaling theory of Anderson localization predicts that all one-electron eigenstates shall be exponentially localized in one-dimensional and quasi-one-dimensional uncorrelated random systems [1]. However, several models with correlated disorder have been introduced in the literature exhibiting a violation of the above scaling prediction to some degree [2–15]. In particular, the presence of short-range dimer-like correlations between the on-site potentials of neighboring sites [2] or between the on-site potential and the transfer integral [3] in one-dimensional systems have been shown to exhibit a resonant energy eigenmode that remains delocalized irrespective of the degree of disorder. The presence of delocalized resonant modes was experimentally demonstrated in dimer-like random semiconductor superstructures [11]. Further, one-dimensional systems with long-range and scale-free disorder have been shown to support mobility edges with a finite energy range of delocalized states [5], a prediction experimentally observed in random microwave guides [14].

Among the low-dimensional models with correlated disorder, a very unique system consists of the diluted Anderson chain [16–22]. The diluted Anderson model was introduced by Hilke [16] and consists of two inter-penetrating sub-lattices,

one composed of random potentials (Anderson lattice) and the other composed of non-random segments of constant potentials. Due to the underlying periodicity, special resonance energies appear which are not affected by disorder. Resonant extended modes have also been reported to appear in binary semiconductor alloys with diluted disorder [17] as well as in random harmonic chains with diluted disorder in the mass distribution [21]. The diluted Anderson model has been extended to include a general diluting function which defines the on-site energies within each non-random segment [18]. The number of resonant extended states was shown to strongly depend on the length of the diluting segments and the symmetries of the diluting function. The extension for a square lattice geometry has shown that this model can exhibit a true metal–insulator two-dimensional (2D) transition with mobility edges delimiting a band of extended states [20].

More recently there has been a growing interest in the study of correlated random systems with a ladder topology [23–30]. Two-channel random ladders have been considered as an interesting class of model systems on which correlations can strongly influence the electronic wavepacket dynamics [31, 32]. In particular, it has been shown that inter-strand and intra-strand correlations favor the wavepacket spreading over longer segments as compared with uncorrelated

random ladders [33, 34]. However, all electronic states remain exponentially localized and, therefore, no long-distance transport is enabled. A quadratic approximation of the two-channel conductance also indicates that the inter-strand correlations and coupling is not enough to promote delocalization [35], although the localization length differs considerably in coding and non-coding segments [36]. Advancing along this line, it has been very recently shown that a quasi-periodic two-chain ladder presents metal/insulator transitions at multiple values of the Fermi energy [37]. In addition, a band of Bloch-like extended states has been demonstrated to emerge when a particular correlation between the on-site potentials and the transfer integral is introduced in a random ladder model [38]. Such extended states co-exist with exponentially localized states, a new scenario that leads to unusual spectral and transport properties [39].

In the present work, we introduce a tight-binding Hamiltonian model of a correlated random ladder. The model incorporates intra-chain base-pairing correlations and diluted inter-chain disorder. We will consider a binary distribution of on-site energies which leads to a density of states with two energy bands in the absence of disorder. Two symmetric resonant delocalized energy states will be shown to persist in the presence of disorder. Their positions will be demonstrated to be related to the energy mismatch of the binary potential. Further, we will show that, although these resonant extended states appear at specific energies and correspond to a null set of the total number of states in the limit of infinite ladders, they lead to an anomalous dynamical behavior. While the wavepacket dispersion displays a super-diffusive spread, its participation function remains finite. This behavior will be related to the emergence of power-law tails in the wavepacket distribution with a super-diffusive cutoff. We report three relevant power-law exponents involved in such super-diffusive dynamics and show that they satisfy a scaling relation.

2. Model and formalism

We will consider a tight-binding model for one electron restricted to move in a two-channel ladder with first-neighbor hoppings. Considering a single orbital per site, the Hamiltonian can be written as:

$$H = \sum_n \epsilon_n c_n^\dagger c_n + V_{\parallel} \sum_n c_n^\dagger c_{n+1} + V_{\parallel} \sum_n c_{n+1}^\dagger c_n, \quad (1)$$

where

$$\mathbf{c}_n = \begin{pmatrix} c_{n,+1} \\ c_{n,-1} \end{pmatrix}, \quad (2)$$

with $c_{n,s}$ and $c_{n,s}^\dagger$ being the usual fermionic creation and annihilation operators acting at site n of the upper ($s = +1$) and lower ($s = -1$) ladder chains. The coupling matrix within each ladder pair is given by

$$\mathbf{\epsilon}_n = \begin{pmatrix} \epsilon_{n,+1} & V_{\perp} \\ V_{\perp} & \epsilon_{n,-1} \end{pmatrix}, \quad (3)$$

where $\epsilon_{n,s}$ is the on-site energy at the n th site in chain s . V_{\perp} corresponds to the inter-chain hopping amplitude between the

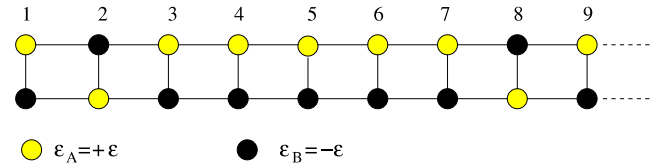


Figure 1. Illustrative representation of the present random ladder model with inter-chain correlations and diluted intra-chain disorder. Each ladder pair is composed of a dimer with distinct on-site energies $\epsilon_A = +\epsilon$ and $\epsilon_B = -\epsilon$. In the odd sub-lattice the dimers have all the same orientation (site A in the upper chain and site B in the lower chain). In the even sub-lattice, a fraction p of the dimers has its orientation reversed (dimers 2 and 8 in this figure).

(This figure is in colour only in the electronic version)

pair of ladder sites at position n . The hopping amplitude along each chain will be considered as site independent and can be written in the form

$$\mathbf{V}_{\parallel} = \begin{pmatrix} V_{\parallel} & 0 \\ 0 & V_{\parallel} \end{pmatrix}. \quad (4)$$

We will restrict our numerical analysis for the particular case of $V_{\parallel} = V_{\perp}$ (equal inter- and intra-strand hopping amplitudes). The general case of $V_{\parallel} \neq V_{\perp}$ does not bring any new qualitative features in what concerns the stationary and dynamical electronic properties. Further, we will use energy units of $V_{\parallel} = V_{\perp} = 1$. The ladder base pair sites will be considered to be composed of dimers with two distinct site energies, which will be taken as $\pm\epsilon$ without any loss of generality. This feature introduces correlations in the energy site distribution of each ladder chain ($\epsilon_{n,+1} = -\epsilon_{n,-1}$) similar to the inter-strand correlations found between the nucleotide base pairs of synthetic poly(C)–poly(G) DNA-like molecules when one considers the energy offset as the average ionization potential of the guanine and cytosine nucleotides. Further, we introduce a diluted disorder distribution of the on-site energies along the ladder chains. In the odd positions along the ladder ($n = 2j - 1, j = 1, 2, 3, \dots$), the dimer pairs will have the same orientation, with the site at the top chain having $\epsilon_{2j-1,+1} = \epsilon$ and the site at the bottom chain having $\epsilon_{2j-1,-1} = -\epsilon$. Disorder will only be present in the sub-ladder corresponding to the even positions ($n = 2j, j = 1, 2, 3, \dots$). In this sub-system, a fraction p of the ladder base pairs will be chosen at random to have its orientation reversed in such a way that $\epsilon_{2j,+1} = -\epsilon$ while $\epsilon_{2j,-1} = \epsilon$. The remaining $(1 - p)$ fraction of base pairs will keep the same orientation of the odd sub-system. Therefore, the model incorporates inter-chain correlations and diluted disorder, ingredients that shall strongly influence the Anderson localization and wavepacket dynamical properties, as we will explore below. Figure 1 shows an illustrative representation of the present ladder model topology on which two base pairs of the even sub-lattice had their dimer orientation reversed.

In the particular case of $p = 0$ the ladder is composed of dimer base pairs with the same orientation. On the other hand, $p = 1$ corresponds to a ladder on which the dimer base pairs have an alternate orientation along the ladder. In these disorder-free limits, the dispersion relation and the resulting

density of states (DOS) can be analytically computed. For the intermediate values of p , the DOS can be numerically obtained through the direct diagonalization of the ladder Hamiltonian.

The presence of disorder leads to the Anderson localization of the energy eigenmodes of quasi-unidimensional systems, except at eventual resonances due to special short-range correlations in the disorder distribution such as dimer-like pairing and dilution. A standard quantity used to quantify the degree of localization is the Lyapunov exponent Γ , which is the inverse of the localization length λ . The Lyapunov exponent can be obtained by exploring the exponential decay of the two-point Green's function as:

$$\Gamma \equiv \frac{1}{\lambda} = - \lim_{N \rightarrow \infty} \frac{1}{N} \ln |G_{1,N}^\dagger(E)|^2, \quad (5)$$

where $G_{1,N}^\dagger$ is the Green's function operator between the first and the last pairs of ladder sites. It can be numerically obtained through a decimation process [40, 41]. The Lyapunov exponent is finite for exponentially localized states as well as outside the energy band. Delocalized or power-law localized states have vanishing Lyapunov exponent in the thermodynamic limit.

The localized/delocalized nature of one-electron eigenstates also influences the electronic wavepacket dynamics. In what follows, we are also going to study the time-evolution of an initially localized wavepacket in the present ladder model. The time-evolution of the wavefunction is obtained from the action of the unitary time-evolution operator:

$$|\psi(\Delta t)\rangle = U(\Delta t)|\psi(0)\rangle = e^{-iH\Delta t}|\psi(0)\rangle, \quad (6)$$

where $|\psi(\Delta t)\rangle$ is the electron state at time Δt , $|\psi(0)\rangle$ is the initial state, and H is the Hamiltonian. We will use a high-order Taylor expansion of the evolution operator

$$U(\Delta t) = \exp(-iH\Delta t) = 1 + \sum_{l=1}^{n_o} \frac{(-iH\Delta t)^l}{l!}. \quad (7)$$

The method can be used recursively to obtain the wavefunction at time t . Our results were taken by using $\Delta t = 0.5$ and the sum was truncated at $n_o = 20$. This cutoff was sufficient to keep the wavefunction norm conservation along the entire time interval considered ($t \leq 10^6$). This formalism is faster than high-order Runge-Kutta methods. We will be particularly interested in calculating the wavepacket dispersion $\sigma(t)$ defined as

$$\sigma(t) = \sqrt{\sum_{s=\pm 1} \sum_{i=1}^N [(i - i_0)^2] |c_{i,s}(t)|^2}, \quad (8)$$

and the participation function $P(t)$

$$P(t) = \frac{1}{\sum_{i,s} |c_{i,s}(t)|^4}, \quad (9)$$

where $c_{i,s}$ are the coefficients of the wavevector expanded in the basis of the Wannier states ($|\psi(t)\rangle = \sum_{i,s} c_{i,s} |i, s\rangle$). Note that $\sigma(t)$ varies from 0, for a wavefunction confined

to the initial base pair, to a maximum value proportional to N , for a wavepacket uniformly extended over the whole chain. The participation function $P(t)$ varies from 1 to N in these same limits [9, 42]. The dispersion is sensitive to the tails of the wavepacket distribution while the participation function measures the number of sites responsible for the major contribution to the particle density. In section 3, we will report our main results for the stationary and dynamical properties of the ladder model with dimer-like correlations and diluted disorder. We will show that the presence of resonant delocalized states associated with the disorder dilution is responsible for an unusual wavepacket dynamics on which the wavepacket dispersion grows super-diffusively while the participation function remains finite. A scaling analysis of the wavepacket distribution will be provided to elucidate the physical origin of the distinct time dependence of the dispersion and participation functions.

3. Results

Let us start by reporting the DOS of the non-random limiting cases of $p = 0$ and 1 together with the most disordered case on which half of the dimers of the even sub-lattice have their orientation reversed at random. In the cases $p = 0$ and 1, the dispersion relation can be analytically computed. For $p = 0$ (all dimers with the same orientation), the mode with positive energy is given by

$$E(k) = 2 \cos k + \sqrt{\epsilon^2 + 1}, \quad (10)$$

where k is the wavenumber along the ladder. For $p = 1$ (dimers having an alternate orientation), the unit cell is composed of a pair of dimers and, therefore, there are two branches of positive energies whose dispersion relations read

$$E(k) = \sqrt{\epsilon^2 + 1 + 4 \cos^2 k} \pm 4 \cos k. \quad (11)$$

From these, the DOS can be directly obtained as $g(E) = \sum [1/(2\pi |dE/dk|)]$, where the sum extends over the eventually degenerated modes. The resulting DOSs for these two cases are shown in figure 2 for the particular value of $\epsilon = 4$. For $p = 0$ the gap between the positive and negative energy bands is $\Delta E = 2\sqrt{\epsilon^2 + 1} - 4 = \simeq 4.25$ while the band width is $\delta E = 4$. For $p = 1$ the gap between the bands becomes much larger $\Delta E = 2\epsilon = 8$ while the energy band becomes narrower $\delta E = \sqrt{\epsilon^2 + 9} - \epsilon = 1$. Actually, the energy band is a superposition of bands originated from distinct modes. An additional Van Hove singularity is present at $E_r = \sqrt{\epsilon^2 + 1} \simeq 4.12$, which delimits the end of one of the superposed bands. This singularity is associated with the additional symmetry of the alternate dimer ladder. It is interesting to notice that this singularity is exactly at the energy corresponding to the band center of the fully oriented dimer ladder ($p = 0$). For the diluted random dimer ladder we obtained the energy eigenvalues by the direct diagonalization of ladders with $N = 10\,000$ base pairs. 100 distinct disorder configurations were considered. The resulting DOS is also reported in figure 2. It was obtained by making the histogram of the energy eigenvalues taking small windows of size $\delta E \simeq$

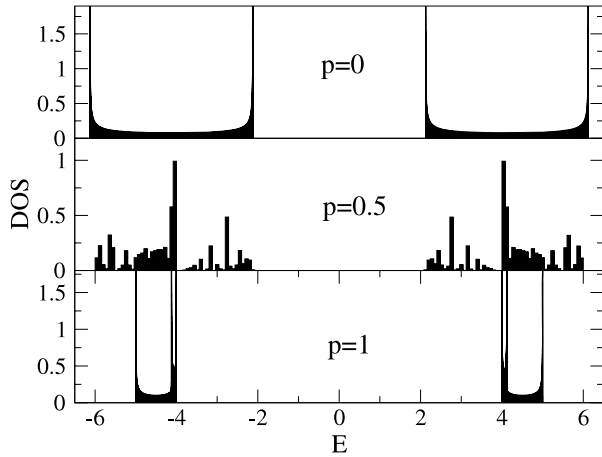


Figure 2. DOS for the tight-binding ladder model with $\epsilon = 4$. Top panel: all base pair dimers have the same orientation ($p = 0$). Bottom panel: the base pair dimers follow an alternate orientation along the ladder sequence. The case $p = 1/2$ corresponds to a ladder with diluted disorder. Dimers at the odd sub-lattice have the same orientation while a fraction $p = 1/2$ of the dimers at the even sub-lattice are chosen at random to have the opposite orientation. Notice that the band gap increases while the band width decreases with p .

0.1. The large fluctuations in the DOS are typical of systems exhibiting Anderson localization due to the effective absence of level repulsion. The Van Hove singularities are rounded due to the loss of translational symmetry introduced by disorder.

In figure 3 we report the spectrum of Lyapunov exponents for the diluted random dimer ladder with $p = 1/2$ and distinct mismatches between the dimer on-site energies. The Lyapunov exponent is mainly finite all over the energy band. This indicates that most of the energy eigenstates are exponentially localized. The typical localization length ($\lambda = 1/\Gamma$) is smaller than 10^2 base pairs. However, a vanishingly small Lyapunov exponent is found at a pair of symmetric resonant energies $\pm E_r$, signaled by a pronounced trough in the Lyapunov exponent spectrum. These resonant modes thus remain extended even in the presence of disorder. They are located at the Van Hove singularities $\pm\sqrt{\epsilon^2 + 1}$, as shown in figure 4. This feature can be understood by stressing that the eigenfunctions corresponding to these resonant modes have null amplitudes at the dimer sites of the random even sub-lattice and, therefore, they are not affected by disorder [43]. Therefore their energies shall consist of the eigenenergies of an isolated dimer, which are actually given by $\pm\sqrt{\epsilon^2 + 1}$. We analyzed the vanishing of the Lyapunov exponent in the vicinity of the resonant mode and found that it scales as $\Gamma \propto |E - E_r|^{1/2}$. It is interesting to notice that a distinct behavior governs the vanishing of the Lyapunov exponent near the resonant mode of the random dimer model on which $\Gamma \propto |E - E_0|^2$ [2, 44–46].

The above results show that the extended modes of the dimer ladder with diluted disorder are located at a pair of symmetric resonant energies. This is in agreement with the result of the one-dimensional counterpart tight-binding model with diluted disorder which has a single resonance. The number of resonances is equal to the number of coupled chains

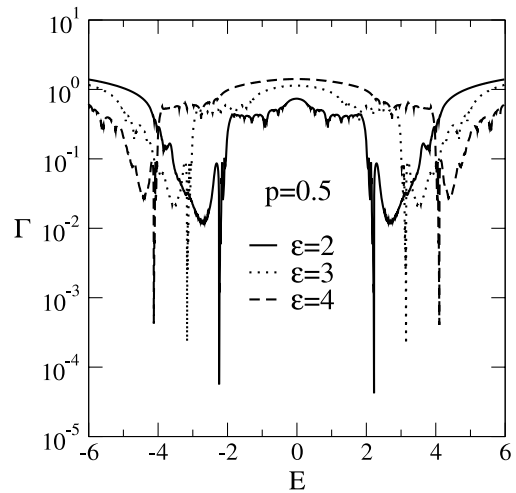


Figure 3. Lyapunov exponent versus energy obtained from the Green’s function decimation on ladders with $N = 10^6$ base pairs. The data are for diluted random ladders with $p = 1/2$ and distinct mismatches of the dimer on-site energies. The Lyapunov exponent vanishes at a pair of symmetric energies, signaled by a dip in the log scale. These correspond to resonant delocalized states that are not sensitive to the underlying diluted disorder.

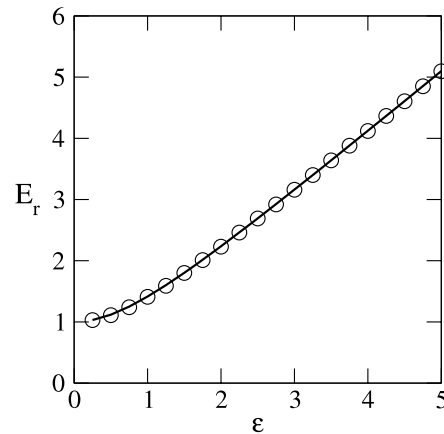


Figure 4. The positive resonant energy corresponding to a delocalized state as a function of the mismatch in the dimer on-site energy ϵ . Symbols were obtained from the numerical computation of the Lyapunov exponent. The solid line gives the positive energy eigenvalue of isolated dimers $E = \sqrt{\epsilon^2 + 1}$.

with diluted disorder. A true band of extended state energies only sets up in the 2D limit [20]. As such, the presence of a finite number of resonances is not able to promote a true metal–insulator transition. However, in what follows, we will show that their presence leads to an unusual time-evolution of a wavepacket initially localized at a single site, chosen to be the central site on the upper leg of the non-random sub-lattice.

Figure 5 reports the time-evolution of the participation function and the wavepacket dispersion. The run time was large enough to surpass any initial transient. No boundary effects are relevant within this run time scale. The participation function saturates after the initial transient showing that the main contribution for the particle probability density comes from a number of sites of the order of 10^1 . This feature

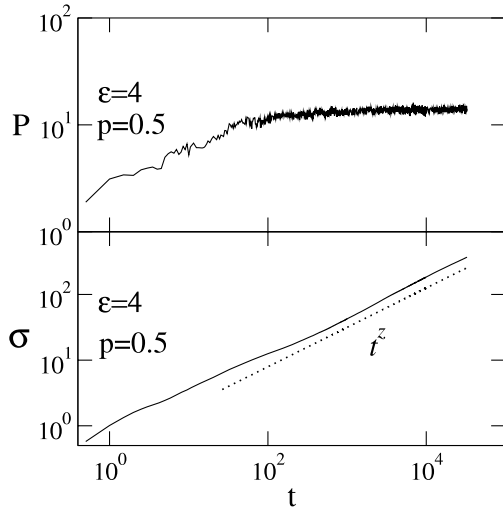


Figure 5. Time-evolution of the participation function and wavepacket dispersion of an initial state localized in a single site of the non-random sub-lattice. The diluted random ladder parameters are $\epsilon = 4$ and $p = 1/2$. The participation function saturates after an initial transient, while the wavepacket dispersion continues to spread super-diffusively. The straight line corresponds to the asymptotic behavior $\sigma \propto t^z$, with $z = 0.60(2)$.

is directly related to the exponentially localized nature of the majority of the one-particle eigenmodes. On the other hand, the wavepacket dispersion displays an asymptotic super-diffusive growth $\sigma \propto t^z$, with $z = 0.60(2)$. This indicates that the wavepacket develops slowly decaying tails that contribute to the dispersion but are not relevant to the wavepacket participation function. The super-diffusive wavepacket spreading in the present ladder model is slower than the one taking place in the random dimer model for which $z = 3/4$ [2, 45, 46]. These distinct super-diffusive spreading laws are associated with the distinct scaling behavior of the Lyapunov exponent on the random dimer and diluted disorder model systems. Some exact results for the characteristic exponents associated with the wavepacket spreading on the random dimer model have been obtained in terms of a scaling scenario [45, 46]. It would be interesting to develop an analogous scaling scenario to provide a deeper understanding of the wavepacket dynamics in the present diluted random ladder model.

In order to have a clearer picture of the wavepacket dynamics, we report in figure 6 the typical wavepacket distribution after a long run. The main frame shows that the main contribution to the wavepacket distribution (averaged over 100 runs on distinct disorder distributions) is indeed concentrated around a small segment of the ladder, in agreement with the measured participation function. In the inset, we show collapsed data of the wavepacket distribution tail at distinct run times. It exhibits a power-law decay up to a cutoff distance x_m from the initial position after which an exponential decay takes place. The cutoff x_m actually delimits the wavepacket front. The data collapse shows that the wavepacket tail obeys the scaling form $|\Psi(t, x)|^2 = t^{-\gamma\phi} f(x/x_m)$, where $x_m \propto t^\gamma$ with $\gamma = 0.80(2)$ and $\gamma\phi =$

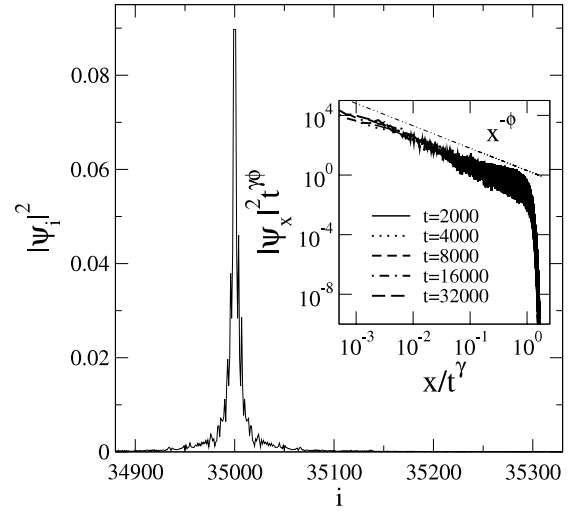


Figure 6. Wavepacket distribution after a long time run averaged over 100 distinct disorder configurations. The initial state was located at the center of a chain with $N = 70\,000$ base pairs. Ladder parameters are the same as in figure 5. Notice that the main contribution for the particle density comes from a narrow range of sites around the initial position. The inset shows collapsed data of the tail of the wavepacket distribution computed at different times. Here $x = i - i_0$. Data are consistent with a universal scaling form $\Psi(x, t) = t^{-\gamma\phi} f(x/x_m)$, where the cutoff distance $x_m \propto t^\gamma$ with $\gamma = 0.80(2)$ and $\gamma\phi = 1.20(5)$. Prior to the ultimate exponential decay, the wavepacket develops a power-law tail $|\Psi|^2 \propto x^{-\phi}$. The measured scaling exponents provide $\phi = 1.50(7)$.

1.20(5). Therefore, the wavefront advances super-diffusively. The power-law tail of the wavepacket distribution is found to scale as $|\Psi(x, t)|^2 \propto x^{-\phi}$, with $\phi = 1.50(7)$.

The above spatial and temporal scaling behavior of the wavepacket distribution is consistent with the distinct dynamical behavior of the participation function and the wavepacket dispersion. In the long time regime, the participation function can be written as:

$$P(t) = P(x_0) + \left[\sum_{x_0}^{x_m(t)} [|\Psi_0|^2 x^{-\phi}]^2 \right]^{-1}, \quad (12)$$

where x_0 is the characteristic distance after which the power-law decay takes place (of the order of a few base pairs) and $|\Psi_0|^2$ is the coefficient of the asymptotic power-law decay of the wavepacket density. Notice that $P(t \rightarrow \infty)$ is independent of the cutoff x_m for $\phi > 1/2$. Therefore the power-law decay of the wavepacket with an exponent $\phi = 3/2$ is fast enough to keep the participation function finite. On the other hand, the wavepacket mean-square displacement can be estimated by

$$\sigma^2(t) = \sigma^2(x_0) + \sum_{x_0}^{x_m(t)} x^2 [|\Psi_0|^2 x^{-\phi}]. \quad (13)$$

This series is convergent only for $\phi > 3$. Therefore, the dispersion in the present case will be sensitive to the wavepacket cutoff. Actually it shall scale as $\sigma \propto x_m^{(3-\phi)/2} \propto t^z$, with $z = \gamma(3 - \phi)/2$. This scaling relation is indeed satisfied by the numerically measured exponents.

4. Summary and conclusions

In summary we introduced a tight-binding model of a correlated dimer ladder with diluted disorder. Each ladder base pair consists of two distinct sites with on-site energies given by $\pm\epsilon$, measured in units of the first-neighbors' hopping amplitude that couples the dimer sites as well as the sites along the ladder chains. The ladder is considered as being composed of two inter-penetrating sub-lattices. In one of them all dimers have the same orientation. In the other sub-lattice, a fraction p of the dimers has the reversed orientation. Therefore, the model combines features present in DNA-like segments, such as base pairing, with those present in low-dimensional systems with correlated disorder, represented by the randomness presence only in a given sub-lattice. We showed that most of the one-particle eigenstates become exponentially localized, except for a pair of symmetric resonant states which are insensitive to the underlying disorder. These states have eigenfunctions with null amplitude at the random sub-lattice and, therefore, their energies correspond to the eigenenergies of an isolated dimer. Such truly delocalized resonant states due to the diluted nature of the disorder are responsible for the unique transport properties of the electronic wavepacket not exhibited by models including just inter- or intra-strand correlations [31, 32, 35, 36].

The resonant extended states were shown to play a relevant role in the one-particle wavepacket dynamics. An initially localized wavepacket evolves in time developing a power-law tail with the wavefront advancing super-diffusively. We found that the wavepacket distribution tail decays with the distance x to the initial position as $|\Psi(x)|^2 \propto x^{-\phi}$, with $\phi = 1.50(7)$. Further, the wavefront super-diffusion is given by $x_m \propto t^\gamma$, with $\gamma = 0.60(2)$. The spatial and temporal scaling of the wavepacket leads to distinct dynamical behaviors for the participation function and the wavepacket dispersion. The exponent ϕ is large enough to keep the asymptotic participation function finite. Therefore this quantity, commonly used to characterize the spatial extension of wavepackets, is not sensitive to the presence of the resonant extended states. On the other hand, the exponent ϕ is not large enough to keep the wavepacket dispersion σ finite. As a consequence, it grows super-diffusively as $\sigma \propto t^z$, with $z = \gamma(3 - \phi)/2 = 0.60$. As such, the wavepacket dispersion captures the existence of the resonant extended modes.

It is interesting to notice that the wavepacket dynamics in nonlinear disordered chains was previously reported to also display a regime on which the wavepacket dispersion diverges while the participation function remains finite [47, 48]. However, this effect has a different physical origin associated with a partial self-trapping of the wavepacket due to the nonlinearity. Here the non-simultaneous divergence of the participation function and dispersion results from the development of power-law tails in the wavepacket distribution. As it is known that the wavefunction in the vicinity of the Anderson transition depicts power-law multifractal tails [49–51], it would be interesting to investigate the possibility of having such non-simultaneous divergence of the participation and dispersion functions at a disorder driven true metal–insulator transition.

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