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# Phase distribution and superstructures on the phase correlation of copropagating electromagnetic fields

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**Abstract.** The phase distribution and phase correlation of two initially coherent electromagnetic field modes copropagating through a lossless nonlinear medium are investigated. We show that the number of distinguishable components in the phase distribution depends on the set of nonlinear parameters through a simple relation and that it is connected with the number of entangled field states as well as the number of components that a single field state acquires after propagating through the medium. The phase correlation between the two field modes is shown to exhibit a rich pattern of collapses and revivals, similar to those observed in the quantum inversion of several generalizations of the Jaynes–Cummings model and is related to beats of the various eigenstates of the total Hamiltonian.

## 1. Introduction

Exactly solvable models of interacting quantum systems have attracted a great deal of interest for over two decades now as they have served as testing ground for fundamental theories of the radiation–matter interaction. In particular, theoretical studies have been centred on quantum systems displaying properties with no classical counterpart. Recent progress in experimental techniques has further enhanced the interest in simple solvable quantum systems once it has been made possible to produce in laboratory idealized physical situations such as two-level atoms [1] and one-photon states [2].

The Jaynes–Cummings [3] model of a two-level atom interacting with a single-mode electromagnetic radiation field is one of the few models that can be solved exactly and yet give non-trivial results, such as the phenomenon of periodic collapses and revivals of the initial atomic population [4]. These collapses and revivals are associated with beats between eigenstates of the total Hamiltonian that oscillate with their respective Rabi frequencies. Between two consecutive revivals, the field and the atom states become almost disentangled, with the field state being, in the large-mean-photon-number limit, a linear superposition of two coherent states [5, 6].

Generalizations of the Jaynes–Cummings model that consider, for example, the interaction of the radiation field with a multilevel atom [7], the system embedded in a nonlinear medium [8], a detuning between field and atom frequencies [7–9], cavity effects [10, 11], and sub-Poissonian field distributions [12] have been exhaustively studied in the literature and exhibit quite similar features. Besides the standard collapse and revivals scenario, some of the generalizations of the Jaynes–Cummings model, particularly those that present a nonlinear spectrum of the Rabi frequencies [8, 9, 12], exhibit a rich structure of fractional revivals and super-revivals after long interaction time intervals, which are associated with the beating of non-nearest-neighbour eigenstates of the system.

The nonlinear interaction of a single-mode radiation field with a Kerr medium can be described, within the adiabatic approximation, by an effective Hamiltonian containing only photon operators [13], whose dynamics can be exactly determined. While preserving the photon number distributions, this effective interaction Hamiltonian can generate a superposition of macroscopically distinguishable coherent states once the interaction time is properly chosen [14, 15]. The generation of a linear superposition of coherent states has also been demonstrated to occur when initially coherent light propagates through a nonlinear birefringent optical medium [16]. Since it was first predicted, the possibility of generating Schrödinger cat-like states using optical fibres has been exploited [17–20]. Miranowicz *et al.* [17] showed that, for an initially coherent field state, the output signal quantum state is composed of $m$ superposed coherent states whenever $\tau = 2\pi(n/m)$, where $n/m$ is an irreducible fraction and $\tau$ is a dimensionless fibre length proportional to the third order nonlinear susceptibility. They also showed that the maximum number of clearly distinguishable components scales as $m_{\max} \propto \bar{n}^{1/2}$, where $\bar{n}$ is the mean photon number. In a realistic experiment, very long interaction times or strong nonlinearities would be required in order to observe these quantum superposition of states. In these cases, dissipative [21] and saturation effects [22] compromise the coherence of the output field states.



The system composed of two single-mode electromagnetic fields copropagating through a lossless Kerr medium is another exactly soluble model with very interesting features. Although the photon statistics of both fields remain unchanged during propagation, the phase dynamics and the squeezing properties are considerable influenced by the coupling between the fields [23]. The entanglement properties of two initially coherent field states were recently investigated [24]. It was shown that the fields reach completely disentangled states whenever $\lambda_i t = 2\pi n$, where $t$ is the interaction time interval and $\lambda_i$ a nonlinear interaction parameter. Such disentangled states can be a well distinguishable Schrödinger cat-like state with $m$ components if $\lambda_a t/4 = \pi(n/m)$, where $\lambda_a$ is the nonlinear self-interaction parameter, provided that $n/m$ is an irreducible fraction and $m < \bar{n}^{1/2}$. Between two consecutive disentanglements the fields reach partial disentanglements. The field state during a partial disentangelement is not a Schrödinger cat-like state but instead a statistical mixture of a number $q$ of states whenever $\lambda_i t = 2\pi(p/q)$, where $p/q$ is an irreducible fraction and $q_{\max} \propto \bar{n}^{1/2}$. In the large-mean-photon-number limit, the purity function is characterized by a multifractal measure whose singularity spectrum exhibits unique features [24, 25].

In this work, we study the phase properties of two initially coherent field modes copropagating in a lossless Kerr medium in the light of the partial disentanglement picture. By using a quantum-mechanical phase distribution formalism to examine the phase configuration of the field state after a finite interaction time interval, we show that the number of distinguishable components in the phase distribution obeys a simple relation between the number of entangled states and the number of components that a single field mode state acquires after propagating through the medium. Therefore the present work reveals the actual composition and degree of coherence of the output field state as a result of self-phase modulation and the two-mode Kerr interaction. Such characterization is an important step through the understanding of interacting field modes as, for example, the field's degree of coherence controls the interference fringes pattern in the probability distribution of a homodyne detector output current. Also, we show that, while the photon number is a constant of motion, phase correlations build up as a signature of the relative entanglement of the field modes. The phase correlation function exhibits a rich pattern of collapses and revivals with superstructures in the long interaction time intervals regime whose origin is the same as those appearing in some generalizations of the Jaynes–Cummings model.

## 2. Phase distribution of entangled field modes

Within the adiabatic approximation, the effective Hamiltonian for two linearly polarized field modes copropagating in a lossless nonlinear medium, interacting through a quantum-optical four-wave mixing mechanism, may be expressed as

$$\mathscr{H} = \omega_a a^\dagger a + \omega_b b^\dagger b + \mathscr{H}_\mathrm{I}, \tag{1}$$

$$\mathscr{H}_\mathrm{I} = \tfrac{1}{4}\lambda_a (a^\dagger a)^2 + \tfrac{1}{4}\lambda_b (b^\dagger b)^2 + \lambda_\mathrm{i} a^\dagger b^\dagger a b, \tag{2}$$

where we used $\hbar = 1$. $a$ ($a^\dagger$) and $b$ ($b^\dagger$) denote the annihilation (creation) operators for the fields $A$ and $B$ respectively, $\lambda_a$ and $\lambda_b$ are related to the third-order nonlinear susceptibility of the medium and $\lambda_\mathrm{i}$ represents the coupling between the two field modes. Note that the mean photon number operators $\hat{n} = a^\dagger a$ and $\hat{m} = b^\dagger b$ are constants of motion and hence there are no oscillations in the mean photon number of each field mode.

Considering that at $t = 0$ both fields are in coherent states $|\alpha_a\rangle$ and $|\alpha_b\rangle$, with $\alpha_a = \bar{n}^{1/2} \exp(\mathrm{i}\xi_a)$ and $\alpha_b = \bar{m}^{1/2} \exp(\mathrm{i}\xi_b)$, the state of the system at time $t$ can be written as

$$|\Psi(t)\rangle = \sum_{n,m=0}^{\infty} a_n b_m \exp(\mathrm{i}\omega_{n,m} t)|n\rangle|m\rangle, \tag{3}$$

with

$$a_n = \frac{\alpha_a^n}{(n!)^{1/2}} \exp\left(-\frac{\bar{n}}{2}\right), \tag{4}$$

$$b_m = \frac{\alpha_b^m}{(m!)^{1/2}} \exp\left(-\frac{\bar{m}}{2}\right) \tag{5}$$

and

$$\omega_{n,m} = \omega_a n + \omega_b m + \tfrac{1}{4}\lambda_a n^2 + \tfrac{1}{4}\lambda_b m^2 + \lambda_\mathrm{i} nm. \tag{6}$$

Describing phase properties has been a long-standing problem in quantum mechanics [26–30], started by Dirac who attempted to define a phase operator via polar decomposition of the annihilation operator [27]. Nowadays, a generalization of Dirac's ideas, introduced by Pegg and Barnett [26], has been used successfully. The formalism consists of decomposing the annihilation operator in a truncated Hilbert space and defining a Hermitian phase operator in this finite-dimensional space. The limit of an infinite-dimensional Hilbert space is made only after all physical quantities are calculated. Shapiro and Shepard [28] showed that the Pegg–Barnett phase operator has the same measurement statistics as the probability operator measure generated by the Susskind–Glogower [29] operator. An equivalent and computationally advantageous method has been proposed by Agarwal *et al.* [30], where, instead of trying to define a Hermitian phase operator, a phase distribution associated with a given density operator $\hat{\rho} = |\Psi\rangle\langle\Psi|$ is introduced as

$$P(\theta_a, \theta_b) = \frac{1}{(2\pi)^2} \langle\theta_a, \theta_b|\hat{\rho}|\theta_b, \theta_b\rangle, \quad (7)$$

where $|\theta_a, \theta_b\rangle$ is the tensor product of coherent phase states $|\theta_b\rangle|\theta_b\rangle$, whose representation in number-ket states $|n\rangle$ is

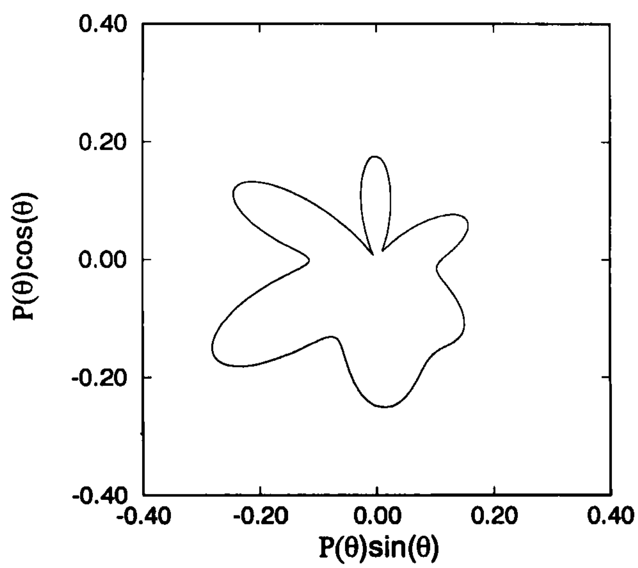
$$|\theta\rangle = \sum_{n=0}^{\infty} \exp(in\theta)|n\rangle. \quad (8)$$

The states $|\theta\rangle$ are the eigenstates of the Susskind–Glogower [29] phase operator and generate the probability operator measure of maximum-likelihood phase estimation [28]. Substituting equations (3)–(6) and (8) in equation (7), and after a short algebraic manipulation, we may obtain the expression for the phase distribution as

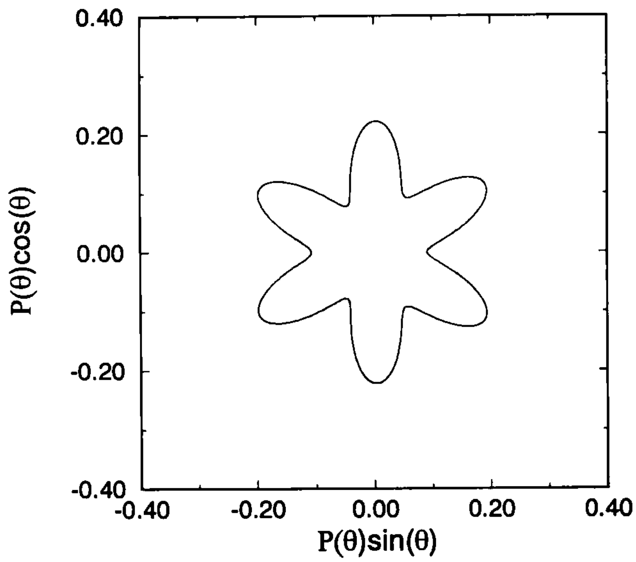
$$P(\theta_a) = \sum_{n,n',m=0}^{\infty} a_n a_{n'}^* b_m b_m^* \exp[\mathrm{i}(n-n')\theta_a - \mathrm{i}(\omega_{n,m} - \omega_{n',m})t], \quad (9)$$

which gives the phase distribution of the optical field $A$ by averaging over all the possible phases of field $B$ ($P(\theta_a) = (1/2\pi)\int d\theta_b\, P(\theta_a, \theta_b)$). For simplicity we have used $\xi_a = \xi_b = 0$ such that the fields are in phase at the entrance of the nonlinear medium. We shall not be interested in the fast free rotation of the phase distribution. The net effect of the nonlinear interactions can then be analysed by defining the phase $\theta = \theta_a + \omega_a t$.

In what follows we investigate the relative role played by both linearly superposed and statistically mixed components of the output field state on the phase probability distribution. In figure 1 we plot some of the field $A$ phase distributions after a propagation time of $\lambda_a t/4 = \pi/6$. If the field $A$ is solely propagating through the medium, it is well known that its final output state is composed, in this case by a linear superposition of six coherent states [17] (figure 1($a$)). Note that large phase uncertainties on each component give rise to




1($a$)

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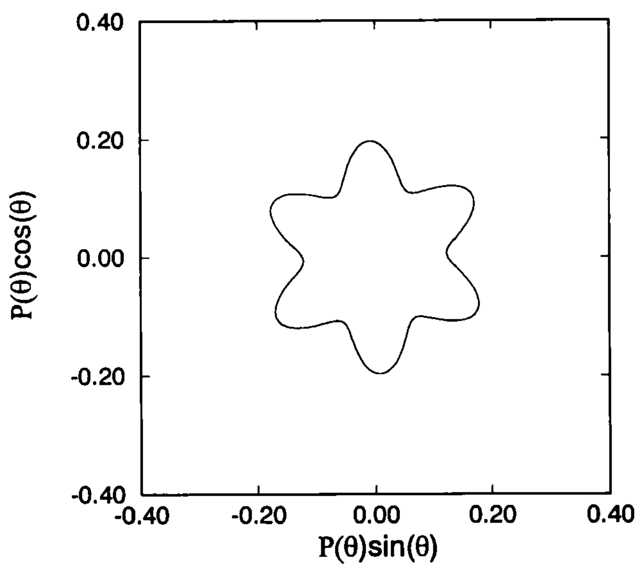



(*c*)

Figure 1. Phase distribution in polar coordinates of an initially coherent electromagnetic field mode with $\bar{n}_a = 3$ after having propagated trough a Kerr medium over a time interval of $\lambda_a t = \pi/6$. The field mode copropagates with a second field mode of mean photon number $\bar{n}_b = 10$ with the interaction nonlinear parameter chosen in such a way that (*a*) $\lambda_i t = 0$, (*b*) $\lambda_i t = 2\pi/3$ and (*c*) $\lambda_i t = \pi/3$.

interference effects which distort the phase distribution. Such interference can be detected as fringes in the probability distribution for a homodyne detector's output current [31]. In figure 1 (*b*) we introduced an interaction between the fields ($\lambda_i t = 2\pi/3$) in such a way that the output field state is composed of a statistical (incoherent) mixture of three states [24]. The phase distribution reveals in this case the presence of six distinguishable components, indicating that each of the three mixed states is composed of a linear superposition (coherent mixture) of two coherent states, that is of a two-component cat-like state. As interference occurs only between coherently mixed components, a less distorted distribution is produced. In figure 1 (*c*) we choose $\lambda_i t = 2\pi/6$ to have the output state as a six-state incoherent mixture. The phase distribution also contains six distinguishable components, indicating that each state in the mixture is a single-component coherent state. Note that the distribution is symmetric as no interference effects come into play among statistically mixed field components. Therefore, in this particular case, the current distribution of a homodyne detector will not exhibit any interference fringe. We further investigated the phase distribution dependence on the set of nonlinear parameters. Our analysis revealed that the total number of components of the output field state is the minimum common multiple (MCM) of the number of statistically mixed states and the number of linearly superposed components that a single field mode acquires after propagating through the Kerr medium.

The maximum number of distinguishable components in the phase distribution scales with $\bar{n}^{1/2}$ once it is intrinsically related to the variance on the phase of each component ($\langle\theta^2\rangle^{1/2} \propto 1/\bar{n}^{1/2}$ and $n_{\max} \sim 2\pi/\langle\theta^2\rangle^{1/2}$). This property makes the above rule for the total number of components difficult to observe numerically whenever the number of predicted components is larger than $\bar{n}^{1/2}$. For large mean photon numbers the phase uncertainty is reduced and the above rule is always precise. Note that in the limit of a very large mean photon number many terms are relevant in the series in equation (9) (of the order of $\bar{n}$). These terms will interfere destructively and give rise to a vanishing probability unless the phase $\theta$ is chosen in such a way that the phase factor $(\omega_{n,m} - \omega_{n',m})t + (n - n')\theta$ satisfies periodic conditions. If one takes $\lambda_a t/4 = \pi p/q$, $\lambda_i t = 2\pi p'/q'$, the above condition is fulfilled whenever $\theta = 2\pi\alpha/\beta$, with $\alpha/\beta$ an irreducible fraction and $\beta$ the MCM of $q$ and $q'$.

## 3. Phase correlation function

During the propagation along the Kerr medium, the electromagnetic fields develop correlations between their phase distributions. These correlations vanish whenever the fields are in completely disentangled states, once a measure over one field cannot interfere in the state of the second field. However, in the general case the field states are entangled and phase correlations are expected. The entanglement between the field states could be measured by detecting photon correlations in a counting experiment [32]. If a photon detector A is open at random, the distribution of elapsed time until the first photon reaches the detector is always flat. However, if the detector A is open just after a photon B is detected, the distribution of elapsed time exhibits a characteristic delay time for phase correlated fields. Among many definitions of the phase correlation, we choose the particular definition

$$\chi_\theta(t) = \langle \exp[i(\theta_a + \theta_b)]\rangle - \langle \exp(i\theta_a)\rangle\langle \exp(i\theta_b)\rangle, \tag{10}$$

as its terms can be easily obtained from equation (7) resulting in the following expressions:

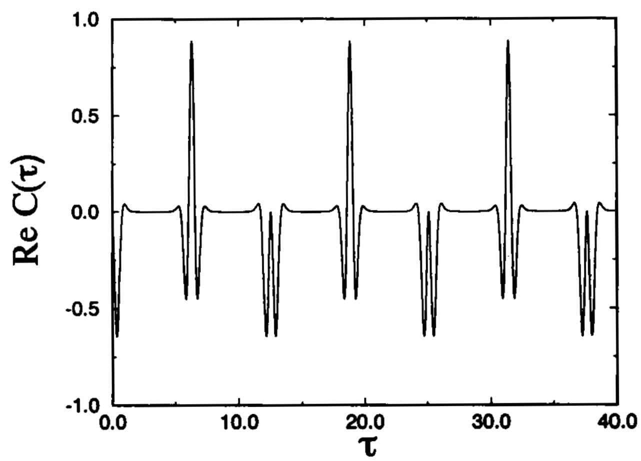
$$\langle \exp[i(\theta_a + \theta_b)]\rangle = \sum_{n,m=0}^{\infty} a_{n+1}a_n^* b_{m+1}b_m^* \exp[i(\omega_{n,m} - \omega_{n+1,m+1})t], \tag{11}$$

and

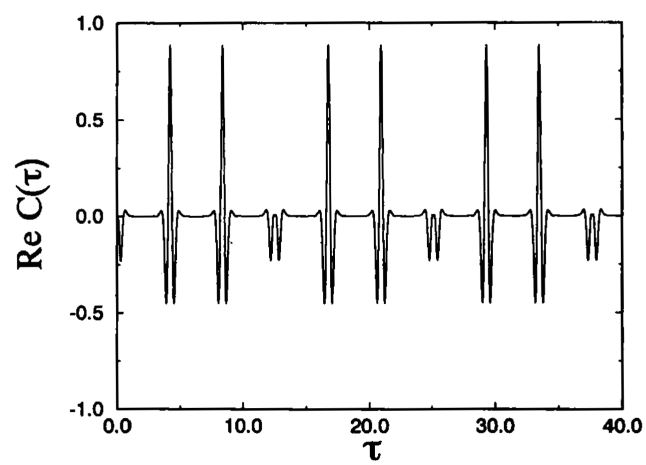
$$\langle \exp(i\theta_a) \rangle = \sum_{n,m=0}^{\infty} a_{n+1} a_n^* b_m b_m^* \exp[i(\omega_{n,m} - \omega_{n+1,m})t]. \tag{12}$$

An analogous expression for $\langle \exp(i\theta_b) \rangle$ is obtained from equation (12) after an appropriate change in indexes.

The above correlation function exhibits a rich temporal evolution pattern. In the following we discuss its behaviour only for the particular cases $\lambda_a = \lambda_b$ and $\bar{n}_a = \bar{n}_b$. Let us study just the behaviour of the function $C(t)$ defined as $\chi_\theta(t) = \exp[-i(\omega_a + \omega_b)t]C(t)$, as it does not contain fast free oscillations. In figure 2 we plot the real part of the correlation function $C(t)$ for two distinct sets of nonlinear parameters. We note that the phase correlation shows a periodic set of collapses and revivals. For $\lambda_i = n\lambda_a$ the phase correlation is a periodic function




(*a*)




(*b*)

Figure 2. The real part of the correlation function $C(\tau = \lambda_a t)$ of copropagating field modes with mean photon number $\bar{n}_a = \bar{n}_b = 1$, $\lambda_a = \lambda_b$, and interaction parameter values of (*a*) $\lambda_i/\lambda_a = 1{\cdot}0$, and (*b*) $\lambda_i/\lambda_a = 2{\cdot}0$.

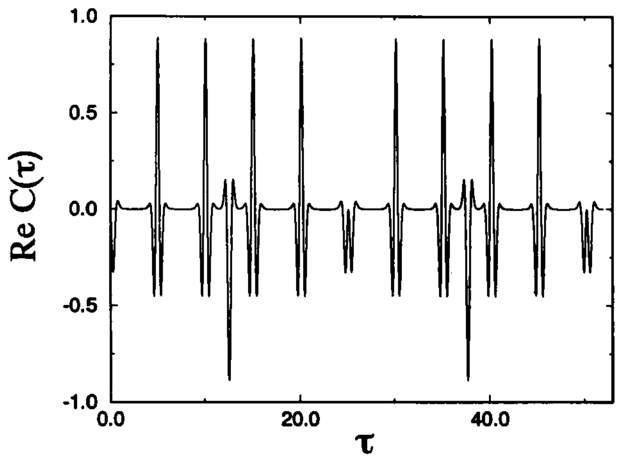



Figure 3. Same as figure 2 with $\lambda_i/\lambda_a = \frac{3}{2}$. Note that the period is twice that obtained in the cases shown in figure 2.

with period $\lambda_a T = 4\pi$, if $n$ is a positive integer. Within each period the phase correlation function has a series of collapses and revivals exhibiting, for small, $n$, $n$ intermediate revivals besides those at the beginning of each new period. If $\lambda_i$ is not an integer multiple of $\lambda_a$, the period of the correlation function becomes $\lambda_a T = 4\pi q$ with $p/q = \lambda_i/\lambda_a$ being an irreducible fraction. The particular case of $q = 2$ ($\lambda_i = \frac{3}{2}\lambda_a$) is shown in figure 3 in order to illustrate the doubling of the period.

A more spectacular series of collapses and revivals takes place whenever the number of intermediate revivals within a period is too large (see figure 4). The correlation function exhibits a very rich modulated structure of revivals, which we shall name hereafter a superstructure owing to its similarity to those appearing in the atomic population of some generalized Jaynes–Cummings models [8, 9, 12]. These superstructures originate from beatings of non-nearest-neighbour eigenstates

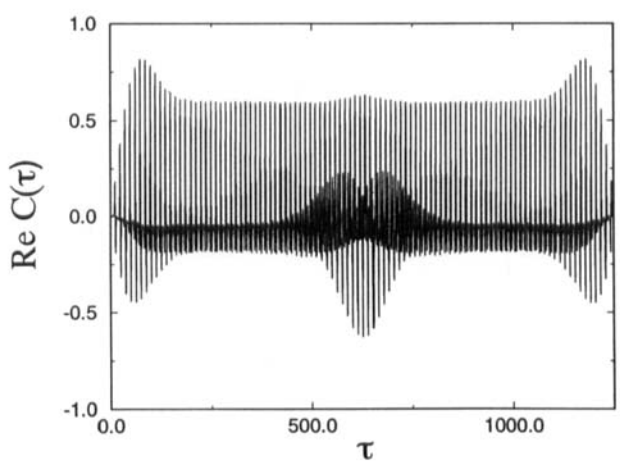



Figure 4. Same as figure 2 but with the interaction parameter $\lambda_i/\lambda_a = 0.01$. The real part of the correlation function exhibits a very long period and symmetric superstructures.

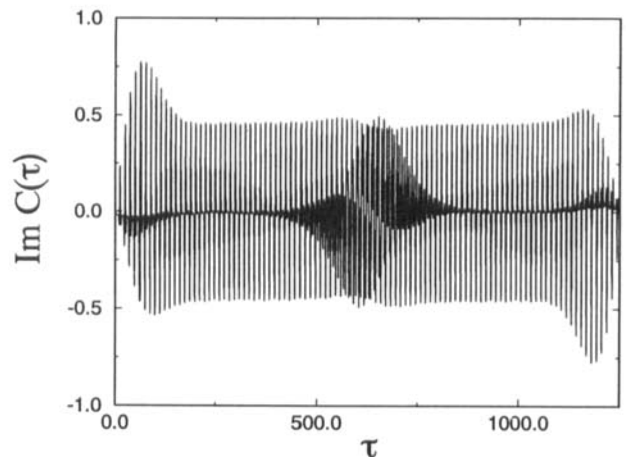



Figure 5. The imaginary part of the correlation function $C(\tau = \lambda_a t)$ for the same parameter set as figure 4. Note that it exhibits an antisymmetric superstructure within each period.

of the total Hamiltonian (see equation (11)). They are more evident in the limit of small mean photon number. This is because, in this limit, only a small number of modes can effectively take part in the evolution.

The main features of the phase correlation presented above remain unchanged when the average photon number $\bar{n}_a \neq \bar{n}_b$ and $\lambda_a \neq \lambda_b$. The imaginary part of the correlation function have similar patterns to those presented by the real part (figure 5). The most significant difference is connected with the antisymmetry presented by the imaginary part within a period [$\text{Im } C(t) = -\text{Im } C(T-t)$], in contrast with the symmetry of the real part of the phase correlation.

## 4. Summary and conclusion

In this work we have studied the phase properties of copropagating electromagnetic fields interacting with a Kerr medium via an effective self-phase modulation term and with each other via a cross-phase modulation mechanism. We showed that the phase distribution of one field mode can be quite modified by the presence of the second field mode. A detailed analysis of the phase distribution indicated that the number of distinguishable components in the phase distribution is the minimum common multiple between the number of components that a single field mode state acquires after propagating through the medium and the number of statistically mixed field states generated after its interaction with the second field mode.

We have also shown that the phase correlation function exhibits a periodic behaviour with a rich pattern of collapses and revivals, with superstructures in the case of long period and small mean photon number. As in some generalized Jaynes–Cummings models, the origin of the superstructures of collapses and revivals comes from the nonlinearity of the spectrum of Rabi frequencies and from the fact that only a small number of eigenstates are sufficiently populated to cause non-negligible contributions in the temporal evolution of the phase correlation.

The effects predicted here can, at least in principle, be observed from the phase correlation of electromagnetic fields copropagating in a Kerr medium such as an optical glass fibre, provided that the fibre is transparent for the input carrier frequencies. However, typical values of the nonlinear third-order susceptibilities require extremely long fibres in order to observe a series of revivals [33]. In this case, dissipative effects become relevant and may destroy the coherence of the output field state [21]. Semiconductor-doped glass fibres [34] and waveguides [35], which have very strong nonlinearities and fast response times [36], would be better suited. However, saturation effects cannot be disregarded in such materials and it is also known that it degrades the field coherence [22]. As only small field intensities are required for the appearance of superstructures, saturation effects may not be relevant. In spite of all such experimental drawbacks which can compromise and obscure the observation of the predicted behaviour, the present results have their own value as they reveal the actual composition of the output quantum state of two electromagnetic field modes due to self-phase modulation effect and the two-mode nonlinear Kerr interaction.



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