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Critical behavior of the two-dimensional Anderson model with long-range correlated disorder

I F dos Santos[1], F A B F de Moura[1], M L Lyra[1] and M D Coutinho-Filho[2]

[1] Instituto de Física, Universidade Federal de Alagoas, Maceió—AL 57072-970, Brazil
[2] Laboratório de Física Teórica e Computacional, Departamento de Física, Universidade Federal de Pernambuco, Recife—PE 50670-901, Brazil

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Abstract

We describe the critical behavior of the Anderson-like transition predicted to occur in the 2D tight-binding model with isotropic scale-free long-range correlated disorder characterized by a power-law spectral density. We explore the scale invariance of the participation function relative fluctuation at the critical point to locate the mobility edge as a function of the power-law spectral density exponent α_{2D}. The states near the band center, which exhibit power-law localization for uncorrelated disorder, become delocalized for $\alpha_{2D} > 2$. In addition, we consider the finite-size scaling hypothesis to estimate the correlation length critical exponent. We find that the critical exponent ν depends on α_{2D}, thus indicating that correlations in the disorder distribution are indeed relevant in this regime, in agreement with the extended Harris criterion.

1. Introduction

The single-parameter scaling hypothesis is the basis of our understanding of the Anderson metal–insulator transition (MIT), a continuous zero-temperature quantum phase transition in disordered systems [1, 2]. The occurrence of a localization–delocalization transition for weak disorder in three-dimensional (3D) geometries, and its absence in low-dimensional systems with time reversal symmetry at any disorder strength, are the most known predictions. However, during the last two decades, it has been shown that low-dimensional disordered systems can support extended states or a localization–delocalization transition in the presence of short-or long-range correlations in the disorder distribution [3–15], as well as long-ranged hopping amplitudes [16–19]. Much attention has been given to the delocalization problem in 1D systems with long-range correlated disorder. It has been reported by several authors [5, 10, 11, 14] that these systems display an Anderson MIT with mobility edges separating localized and extended states for sufficiently strong correlations. In particular, the effect of long-range correlated scattering on the transport properties of microwave guides was studied and corroborated the predicted presence of mobility edges [15].

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Recently, the critical behavior of the Anderson transition in the presence of scale-free disorder in 3D was the subject of a detailed investigation [20]. It was found that, for weak correlations, the localization length exponent remains that of the uncorrelated system. On the other hand, it depends on the correlation exponent for strong correlations, in agreement with the extended Harris criterion [21] (concerning the original Harris criterion see, e.g., [22, 23]). The extended Harris criterion asserts that $1/r^{\gamma}$ decaying correlations in the disorder distribution will be irrelevant to the critical behavior if the correlation length critical exponent ν_0 of the transition occurring in the absence of correlations satisfies either the inequality $\nu_0 > 2/d$ for correlations with $\gamma > d$ or $\nu_0 > 2/\gamma$ for correlations with $\gamma < d$. In contrast, correlations become relevant and a new long-range disorder fixed point becomes stable with the correlation length exponent assuming non-universal values given by $\nu = 2/\gamma$. The extended Harris criterion was developed for systems presenting a second-order phase transition in the presence of uncorrelated disorder. Therefore, it may not directly apply for systems presenting a correlation-induced transition. Nevertheless, using the hypothesis that the normalized localization length obeys a single-parameter scaling close to the correlation-induced transition in the 1D Anderson model, the localization length critical exponent was computed [14] and found to depend on the exponent of the disorder spectral density.

A first study of the effects of long-range correlations in the localization properties of 2D electronic systems with orthogonal symmetry was performed in [24]. The authors considered a striped medium in the x–y plane with on-site disorder. The on-site energies were generated by a superposition of an uncorrelated term and a long-range correlated one along the y-direction. It was predicted that this system displays a disorder-driven Kosterlitz–Thouless MIT in the regime of strong correlations. More recently, the effects of long-range correlations in *both* x and y directions were studied [12, 25, 26]. A transfer matrix numerical calculation on a striped geometry, combined with finite-size scaling arguments, confirmed the presence of a correlation-induced Kosterlitz–Thouless transition [25]. In addition, by considering the site energies of the 2D Anderson Hamiltonian distributed in such a way as to have a power-law spectral density $S(k) \propto 1/k^{\alpha_{2D}}$, an exact diagonalization formalism of finite lattices with a square geometry showed that for $\alpha_{2D} > 2$ this model displays a phase of low-energy extended states. In this regime, the dynamics associated with the spread of an initially localized wavepacket becomes ballistic [12]. Moreover, the exponents governing the collapse of the participation function for low energies ($\xi \propto L^{D_2}$) and the long-time decay of the autocorrelation function [$C(t) \propto t^{-\beta}$] were shown to satisfy the scaling relation $D_2 = \beta d$. More recently, Bloch oscillations were predicted to occur in the $\alpha > 2$ regime, whose amplitudes are consistent with the width of the band of extended states, according to a phenomenological semi-classical approach [26].

In this work, we report further progress along the above lines made by performing a scaling analysis of the critical behavior of the 2D Anderson model in the presence of isotropic scale-free long-range correlated disorder. The potential landscape will be generated to have a spectral density decaying as $1/k^{\alpha_{2D}}$ which implies its scale-free character. In what follows, we use an exact diagonalization algorithm to compute the participation function which is usually taken as an important tool for measuring the localized/delocalized nature of the energy eigenstates. We will assume a single-parameter scaling hypothesis to hold for the relative fluctuation of the participation function to obtain the phase diagram and to estimate the critical correlation length exponent. In agreement with the 1D behavior, the width of the extended phase is shown to saturate for strong degrees of correlations $\alpha_{2D} \to \infty$. However, for weak correlations the states near the band center remain critical-like, exhibiting power-law scaling. The estimated values for the correlation length exponent are shown to depend on the power-law spectral density exponent, which indicates the relevance of the long-range character of the disorder.

2. Model and scaling assumptions

We consider the 2D Anderson Hamiltonian with disordered on-site energies ϵ_{im} on a regular $L \times L$ lattice [2, 24]:

$$H = \sum_{i,m} \epsilon_{im} |i,m\rangle\langle i,m| + t \sum_{\langle im,jn\rangle} (|i,m\rangle\langle j,n|), \tag{1}$$

where $|i,m\rangle$ is a Wannier state localized at site (i,m) and $\sum_{\langle im,jn\rangle}$ represents a sum over nearest-neighbor pairs. In our calculations, we fix the energy scale by setting the hopping energy $t = 1$. In order to generate a long-range correlated on-site energy landscape, we apply a 2D discrete Fourier transform method defined by

$$\epsilon_{i,m} = \sum_{k_i=1}^{L/2} \sum_{k_m=1}^{L/2} \frac{\zeta(\alpha_{2D})}{(k_i^2 + k_m^2)^{\alpha_{2D}/4}} \times \cos\left(\frac{2\pi i k_i}{L} + \psi_{i,m}\right) \cos\left(\frac{2\pi m k_m}{L} + \phi_{i,m}\right), \tag{2}$$

where $\psi_{i,m}$ and $\phi_{i,m}$ are $L^2/2$ independent random phases uniformly distributed in the interval $[0, 2\pi]$ and $\zeta(\alpha_{2D})$ is a normalization constant which is chosen to have the energy variance $\langle \varepsilon_{i,m}^2 \rangle = 1$. We also shift the on-site energies in order to have $\langle \varepsilon_{i,m} \rangle = 0$. Typically, this sequence is the trace of a 2D fractional Brownian motion [27] with a well defined power-law spectrum $S(k) \propto 1/k^{\alpha_{2D}}$, where $k = \sqrt{k_i^2 + k_m^2}$.

The case of $\alpha \to \infty$ corresponds to a tight-binding model with a harmonic potential of wavelength L. In this limit most of the states become extended due to the effective absence of disorder except for a few states trapped at the system boundaries. In the opposite limit of $\alpha = 0$ one recovers the Anderson model with uncorrelated disorder. The nature of the eigenstates of the 2D Anderson model with uncorrelated disorder has been a subject of intensive investigations due to the unconventional behavior predicted by the scaling theory of Anderson localization [31–34]. On the basis of the scaling theory for the Anderson transition, all states are expected to be exponentially localized. However, the scaling hypothesis leads to the prediction that the localization length of the low-energy states will be macroscopically large, especially at low and intermediate disorder strengths. Further, at scales smaller than the localization length, the eigenfunctions exhibit multifractal fluctuations typical of critical states having a power-law envelope. Therefore, in numerical simulations of finite systems, a *pseudo-mobility edge* is observed when varying the disorder strength or the energy, delimiting a transition from power-law localized to exponentially localized states [31]. Recently, the power-law finite-size scaling was reported to hold even in the presence of correlated disorder [12]. It was also shown that the scaling relation between the exponents of the localization length and the long-time decay of the autocorrelation function remains valid in the power-law regime. Once the numerical simulations are restricted to relatively small lattices, the scaling prediction that a crossover to exponential localization will take place at very large lattices cannot be probed numerically. However, it is important to stress here that a recent reanalysis of scaling arguments has suggested that the power-law localized regime may hold even in the thermodynamic limit [35].

In order to investigate the physical properties associated with the nature of one-electron eigenstates ($|\Phi^u\rangle$), we numerically diagonalize the Hamiltonian and calculate the participation function $\xi(u)$ defined by [2]

$$\xi(u) = \frac{\sum_{i,m} |c_{i,m}^{(u)}|^2}{\sum_{i,m} |c_{i,m}^{(u)}|^4}. \tag{3}$$

Here, $c_{i,m}^{(u)}$ are the amplitudes of the eigenstate $|\Phi^{(u)}\rangle$ in the Wannier representation ($|\Phi^{(u)}\rangle = \sum_{i,m} c_{i,m}^{(u)} |i,m\rangle$). In general the participation number is a good estimate of the number of sites

that participate in the wavefunction. For extended states, ξ is proportional to the total number of sites ($\xi \propto L^2$ for a square lattice). On the other hand, wavefunctions presenting power-law decaying tails may display an anomalous scaling of the participation number $\xi \propto L^{D_2}$, with $D_2 < d$ [28]. We averaged ξ in a small window ΔE around E:

$$\langle \xi(E) \rangle = \left[\sum_{u=E-\Delta E/2}^{u=E+\Delta E/2} \xi(u) \right] \Big/ N_{\mathrm{E}}. \tag{4}$$

We use $\Delta E \approx 0.1$ and a large number of samples such that the number of eigenmodes at each window (N_{E}) is close to 10^5 in order to obtain good accuracy. Here, we will be particularly interested in computing the fluctuation of the participation number defined by

$$\Delta\xi(E) = \sqrt{\langle \xi(E)^2 \rangle - \langle \xi(E) \rangle^2}, \tag{5}$$

where $\langle \xi(E)^2 \rangle$ can be computed as in equation (4). The relative fluctuation of the participation number is given by

$$\eta(E) = \Delta\xi(E)/\langle \xi(E) \rangle. \tag{6}$$

Within the framework of random and non-random long-range hopping models, it was demonstrated rigorously that the distribution function of the participation function is scale invariant at the Anderson transition [29]. Such scale invariance has been used to monitor the critical point of long-range hopping models [30] and will also hold for general models exhibiting a localization–delocalization transition. Here, we will use this feature to obtain the phase diagram of the 2D long-range correlated Anderson model by exploring the fact that the relative fluctuations of the participation function will be size independent at the critical point.

For extended states, the relative fluctuation $\eta(E)$ vanishes continuously with increasing system size since the participation function $\xi(E)$ diverges linearly while the fluctuation $\Delta\xi(E)$ has a weaker size dependence resulting from self-averaging. In the opposite regime of exponentially localized states, the relative fluctuation grows with increasing system size converging to a finite value. The above features can also be described in a scaling form which provides a useful tool for locating the critical point and for measuring the correlation length exponent. Within the framework of the single-parameter scaling hypothesis, the relative fluctuation η in the vicinity of the mobility edge can be written in the form

$$\eta(E) = g[(E - E_{\mathrm{c}})L^{1/\nu}], \tag{7}$$

which reflects the scale invariance of the participation number distribution at the critical point on which the relative fluctuation assumes the value $g(0)$ irrespective of the system size. For localized ($E > E_{\mathrm{c}}$) and extended ($E < E_{\mathrm{c}}$) states the relative fluctuation becomes size dependent. In the thermodynamic limit, it converges to $g(+\infty) =$ constant for localized states while it goes to $g(-\infty) = 0$ for extended states. The relative fluctuation thus converges to a step function as $L \to \infty$, with a discontinuity at the mobility edge. Therefore, when plotting the relative fluctuation as a function of the mode eigenenergy E, the curves obtained from different chain sizes will cross roughly at a single point identifying the mobility edge (actually at $\pm E_{\mathrm{c}}$ in the present model). A small spread of the crossing point is usually due to small corrections to scaling that are present in numerical calculations on finite systems. The above scenario is valid whenever critical states appear at a single energy. For systems supporting a phase of critical states, the relative fluctuation of the participation number remains scale independent over the entire critical phase. In addition, we will employ a finite-size scaling analysis to estimate the correlation length exponent ν which governs the scaling behavior of the relevant length scale in the vicinity of the mobility edge, i.e., $l_\infty \propto |E - E_{\mathrm{c}}|^{\nu}$. According

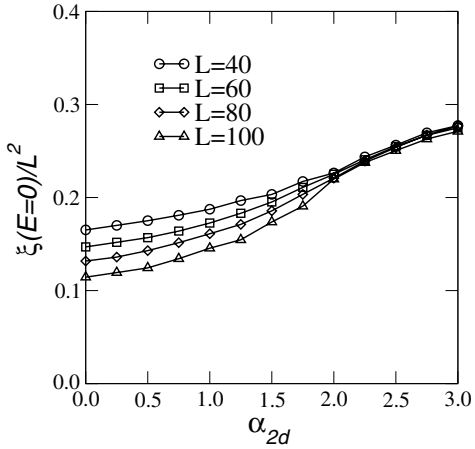

Figure 1. Scaled participation function $\xi(E = 0)/L^2$ versus α_{2D}, for lattices with $L^2 = 40^2, 60^2, 80^2, 100^2$ sites. For $\alpha_{2D} > 2$ the scaled participation function $\xi(E = 0)/L^2$ is size independent, signaling the presence of extended states.

to the above scaling hypothesis, the derivative $\delta = \partial\eta(E)/\partial E$ will scale in the vicinity of the critical point as

$$\delta = \partial\eta(E)/\partial E = L^{1/\nu} f[(E - E_c)L^{1/\nu}]. \tag{8}$$

At the mobility edge $E = E_c$, δ scales with the system size as a power law $L^{1/\nu}$ from which the correlation length exponent ν can be directly estimated.

3. Phase diagram and critical behavior

We collected in figure 1 results for the scaled participation function near the band center $\xi(E = 0)/L^2$ versus the exponent α_{2D} of the power-law spectral density of the correlated potential for lattices with $L^2 = 40^2, 60^2, 80^2, 100^2$ sites. For $\alpha_{2D} > 2$ the scaled participation function $\xi(E = 0)/L^2$ becomes size independent, i.e. the participation function is proportional to the total number of sites. This feature is a clear signature of the existence of extended sates at the band center for $\alpha_{2D} > 2$. In order to locate the mobility edge within the energy band, we employed a scaling analysis of the relative fluctuation of the participation function. In figure 2 we show the relative fluctuation of the participation number $\eta(E)$ versus energy E for $\alpha_{2D} = 1.5$ and lattices with $L^2 = 40^2, 50^2, 60^2, 70^2$ sites. Within our numerical accuracy, one can observe a quite well defined collapse near the band center. At the bottom and top of the band, the relative fluctuation increases with the system size due to the exponential localization of the eigenstates. The collapse in the low-energy region is associated with the nonlinear finite-size scaling previously found for the participation number in this regime [12]. It corresponds to the energy range for which the eigenstates show multifractal fluctuations (critical-like character) at the scaling length of the system sizes used, as is reported to occur in the 2D Anderson model with uncorrelated disorder [31]. In figure 3, we show data for the relative fluctuation of the participation number versus energy E for $\alpha_{2D} = 2.5$ with $L^2 = 40^2, 50^2, 60^2, 70^2$ sites. One can observe a well defined crossing signaling the mobility edge (see the inset). The present estimate for $|E_c| = 2.9$ is consistent with the range of energies with linear scaling of the participation function with increasing system size [12]. For $|E| < E_c$, the relative fluctuation decreases while increasing the system size due to the extended nature

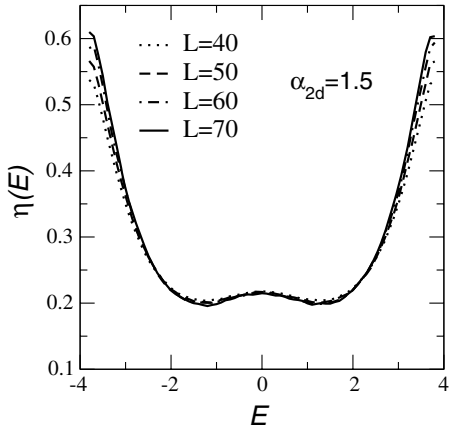

Figure 2. The relative fluctuation of the participation function $\eta(E) = \Delta\xi(E)/\langle\xi(E)\rangle$ versus energy E for $\alpha_{2D} = 1.5$, with $L^2 = 40^2, 50^2, 60^2, 70^2$ sites. Note that near the band center, $\eta(E)$ is size independent over a finite energy range, indicating the power-law scaling of the states in this region.

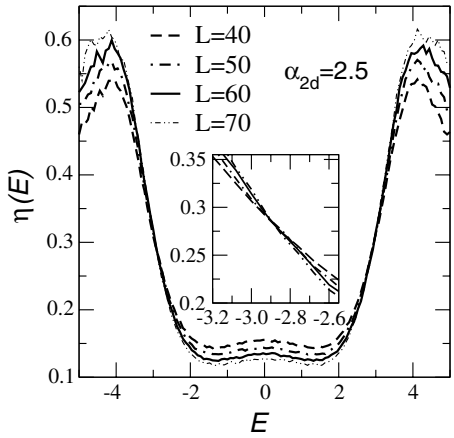

Figure 3. The relative fluctuation of the participation function $\eta(E) = \Delta\xi(E)/\langle\xi(E)\rangle$ versus energy E for $\alpha_{2D} = 2.5$, with $L^2 = 40^2, 50^2, 60^2, 70^2$ sites. The scale invariant point signals the transition between extended and exponentially localized states (see the inset). The crossing point gives the mobility edge $|E_c| = 2.9$.

of the eigenstates, as pointed out in section 2. It increases with L for $|E| > E_c$ indicating exponentially localized states.

In figure 4, we show the phase diagram for this model. We can see that, for strong degrees of correlation ($\alpha_{2D} \to \infty$), the width of the extended band saturates. In contrast with the case for the phase diagram for the 1D counterpart model [5], the width of the extended band seems not to vanish for $\alpha \to 2$. Indeed, the states near the band center exhibit power-law scaling for the system sizes investigated [12], i.e. a typical behavior of critical states. It is important to stress here that the present simulations on finite lattices cannot rule out the possibility of a crossover from critical-like states to exponential localization at macroscopically large lattice sizes [32]. Moreover, the present phase diagram was obtained assuming that the disorder width is of the same order as the typical energy scale given by the hopping amplitude. For sufficiently

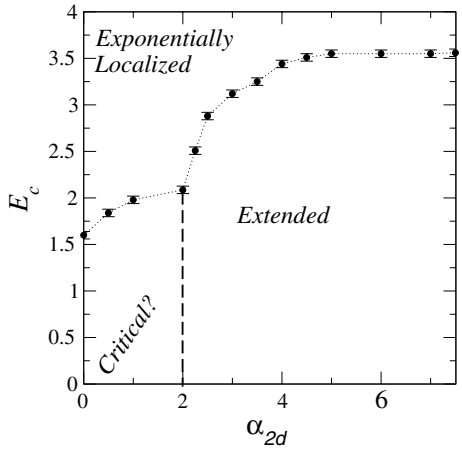

Figure 4. Phase diagram in the (α_{2D}, E_c) plane. The phase of extended states emerges for $\alpha_{2D} > 2$ whose width saturates as $\alpha_{2D} \to \infty$. For $\alpha_{2D} < 2$ the states near the band center remain critical within the system sizes considered. The present data cannot be used to rule out the possibility that these critical-like states may develop an exponential decay for much larger system sizes.

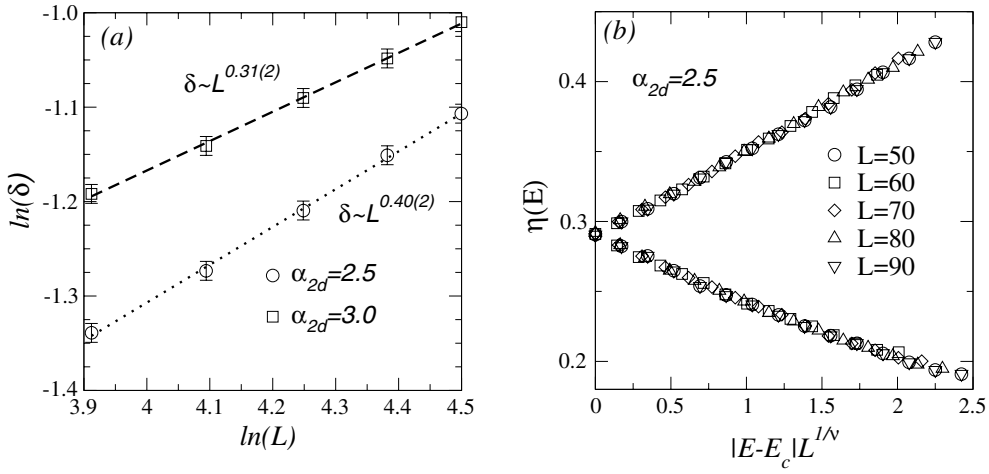

Figure 5. (a) The derivative $\delta = \partial\eta(E)/\partial E$ at the mobility edge versus L for $\alpha_{2D} = 2.5$ and 3.0. The solid lines correspond to power-law fittings $\delta \propto L^{1/\nu}$. No corrections to scaling were considered. (b) The relative fluctuation of the participation function $\eta(E)$ near the critical energy for $\alpha_{2D} = 2.5$ versus the scaling variable $|E - E_c|L^{1/\nu}$, with $\nu = 2.5$. All data collapse to a universal scaling form indicating the accuracy of the estimated correlation length exponent. Upper and lower branches correspond, respectively, to exponentially localized and extended states.

strong disorder strengths, the states at the band center will remain exponentially localized regardless of the value of the exponent α, as is reported to occur in the one-dimensional version [14].

In figure 5(a) we show our numerical estimates for the derivative $\delta = \partial\eta(E)/\partial E$ at the critical point, as computed from two distinct spectral density exponents α_{2D}. These data are consistent with the finite-size scaling hypothesis with a single length scale, equation (8), which predicts a simple power-law size dependence governed by the correlation length exponent ν.

From the power-law fittings, we obtained the exponent $\nu = 2.5 \pm 0.1$ for $\alpha_{2D} = 2.5$ and $\nu = 3.2 \pm 0.2$ for $\alpha_{2D} = 3.0$. For larger values of α_{2D}, the numerical estimate of the derivative becomes less certain once the mobility edge approaches the band edge and the vanishing small density of states near the extremal of the energy band degrades the statistical averages, thus resulting in larger error bars. Using the estimated correlation length exponent, we report in figure 5(b) data for $\alpha_{2D} = 2.5$ in a universal scaling form, with $L^2 = 50^2, 60^2, 70^2, 80^2$ and 90^2 sites (same lattice sizes as in figure 5(a)). The upper branch corresponds to the phase of exponentially localized states while the lower branch corresponds to the low-energy extended phase. The fact that data from distinct lattices sizes fall on a single curve, without the need for any additional adjusting parameter, reflects the accuracy of the estimated correlation length exponent and the absence of significant corrections to scaling in the asymptotic regime investigated.

4. Conclusions

In this work we considered the 2D Anderson model with correlated disorder characterized by the spectral density $S(k) \propto 1/k^{\alpha_{2D}}$. Using an exact diagonalization formalism on square lattices and finite-size scaling analysis, we investigated the participation number and its relative fluctuation along the entire energy band. For $\alpha_{2D} > 2$, we obtained a well defined crossing of the relative fluctuation of the participation number at $E = E_c$, locating the mobility edge separating extended and exponentially localized states. For $|E| < E_c$ the relative fluctuations decrease as L increases. This is a clear signature of extended states. However, for $|E| > E_c$ the relative fluctuation increases with system size, indicating exponentially localized states. For $\alpha_{2D} < 2$ our data have the signature of a transition from exponentially localized to critical-like states. This finding is consistent with the well defined crossover from exponentially localized to power-law localized states observed to occur in the uncorrelated 2D Anderson model on finite lattices [31–33]. The present results cannot be used to distinguish between the scenario derived from the single-parameter scaling theory which states that these critical-like eigenfunctions will exhibit a crossover to exponential localization at very large system sizes [32] and the recent proposal that these states may remain critical in the thermodynamic limit [35]. We provided the full phase diagram in the $\alpha_{2D} \times E$ plane. Using a finite-size scaling hypothesis, we obtained a very nice data collapse from different system sizes close to the critical point of the Anderson transition in the regime of $\alpha_{2D} > 2$. The critical correlation exponent was estimated to be $\nu = 2.5 \pm 0.1$ for $\alpha_{2D} = 2.5$ and $\nu = 3.2 \pm 0.2$ for $\alpha_{2D} = 3.0$, indicating that the correlation length exponent is non-universal. It is important to discuss the above results in the light of the extended Harris criterion for the possible relevance of long-range correlations in disordered systems presenting a second-order phase transition [21]. In the present case, the transition between critical-like and exponentially localized states observed is effectively described as a Kosterlitz–Thouless transition [31, 35], which has a diverging localization length exponent. The extended Harris criterion thus leads to the prediction that long-range correlations will be irrelevant to the critical behavior. This feature is indeed observed for all values of $\alpha_{2D} < 2$, for which the phase transition remains from critical-like to exponentially localized states. It is interesting to note that this is the range of α_{2D} values for which the random potential presents anti-persistent increments [5] and includes all statistically stationary potential landscapes that have $0 \leqslant \alpha_{2D} < 1$ [36]. The transition from exponentially localized to extended states observed for $\alpha_{2D} > 2$ is not anticipated from the extended Harris criterion, although the relevance of long-range correlations to disorder in this regime is indeed reflected by the non-universal behavior of the localization length exponent.

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