Eur. Phys. J. B **80**, 59–63 (2011)
DOI: 10.1140/epjb/e2011-20016-3

THE EUROPEAN
PHYSICAL JOURNAL B

Regular Article

Extended acoustic waves in diluted random systems

A.E.B. Costa and F.A.B.F. de Moura[a]

Instituto de Física, Universidade Federal de Alagoas, Campus A. C. Simões, BR 104 km 14, Tabuleiro dos Martins, 57072-970 Maceió AL, Brazil

Received 9 January 2011 / Received in final form 4 February 2011
Published online 9 March 2011 –

Abstract. In this paper we study the propagation of acoustic waves in an one-dimensional diluted random media which is composed of two interpenetrating chains with pure and random elasticity. We considered a discrete one-dimensional version of the wave equation where the elasticity distribution appears as an effective spring constant. By using a matrix recursive reformulation we compute the localization length within the band of allowed frequencies. In addition, we apply a second-order finite difference method for both time and spatial variables, and study the nature of the waves that propagate in the chain. We numerically demonstrate that the diluted random elasticity distribution promotes extended acoustic modes at high-frequencies.

1 Introduction

The wave propagation in low-dimensional disordered systems is a well known issue with several connections with electronic eigenstates [1–4], magnetic excitations [5,6], vibrational modes [7,8], acoustic wave propagation [9–19] and light propagation [20–23]. The step ahead concerning the transport properties in non periodic systems was done by Anderson in its well known "localization theory" [2,4]. By considering an one-electron Hamiltonian it was predicted the absence of extended eigenstates in low-dimensional systems with uncorrelated disorder [2,4]. In a three-dimensional lattice, the presence of weak disorder promotes the localization of the high-energy eigenmodes. The prediction of exponential localization of all one-electron eigenfunctions in 1D systems can be violated when special short-range [24–31] or long-range [32–35] correlations are present in the disorder distribution. From the experimental point of view, these theoretical predictions were useful to explain transport properties of semiconductor super lattices [31] and microwave transmission spectra of a single-mode waveguide with intentional correlated disorder [35].

Within the context of 1D disordered models with resonant propagation, the Anderson model with "diluted disorder" has attracted a renewed interest [36–42]. The kind of short-range correlation present in the diluted on-site energy disorder distribution was pointed as a key mechanism to promote extended electronic eigenstates [36–42]. Hilke [36] introduced an Anderson model with diagonal disorder diluted by an underlying periodicity. The model consists of two interpenetrating sub-lattices, one composed of random potentials (Anderson lattice) and the other composed of non-random segments of constant potentials. Due to the periodicity, special resonance energies appear with vanishing wave-function amplitudes on the random lattice. The extended states in the diluted Anderson model are in fact Bloch waves with infinite coherence length. In reference [38], the authors presented a simple model for alloys of compound semiconductors by introducing an one-dimensional binary random system where impurities are placed in one sub lattice while host atoms lie on the other sub lattice. The existence of an extended state at the center of the band was demonstrated, both analytic and numerically. Moreover, it was numerically demonstrated that the diluted random Anderson chain displays a nontrivial electronic dynamics [42]. It was reported a long-time memory effect which is reflected by distinct asymptotic dynamics governing the wave-function spreading for electrons initially localized at random or pure sites [42]. Recently, the diluted Anderson model was extended for a 2D lattice [43]. Using analytical and numerical methods, it was shown that the 2D diluted Anderson model displays a metal-insulator transition which was compared with recent experimental results in 2D disordered samples [44,45].

In this letter, we study the acoustic waves propagation in low-dimensional systems with diluted disorder distribution. We considered a discrete one-dimensional version of the wave equation where the elasticity distribution appears as an effective spring constant [13]. The system is composed of two interpenetrating chains with pure and random elasticity intensity. By using a matrix recursive reformulation we compute the localization length within the band of allowed frequencies. In addition, we apply a second-order finite difference method for both time and spatial variables, and study the nature of the waves

[a] e-mail: fidelis@if.ufal.br

that propagate in the chain. We numerically demonstrate that the diluted random elasticity distribution promotes a high-frequency extended acoustic mode with divergent localization length.

2 Model and formalism

The acoustic wave equation in a random media is given by (see Ref. [13]):

$$\frac{\partial^2}{\partial t^2}\psi(x,t) = \frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x,t)}{\partial x}\right]. \quad (1)$$

Where, $\psi(x,t)$ is the wave amplitude, t is the time, and $\eta(x) = e(x)/m$ is the ratio of the stiffness $e(x)$ and the medium mean density m. We consider the wave amplitude with a time-dependent harmonic form $\psi(x,t) = \psi(x)\exp(-i\omega t)$, where ω is the wave frequency. We will use a finite-difference (FD) method to write the acoustic wave equation in a discretized form. The spatial wave amplitude $\psi(x)$ is written as ψ_i where $x = i\Delta x$. The spatial derivative will be written as $(\partial\psi(x))(\partial x) \approx (\psi_i - \psi_{i-1})/\Delta x$. Following reference [13] we will use $m = 1$ and consider nearest-neighbor spacing $\Delta x = 1$. Therefore, the right side of equation (1) can be written as

$$\frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x)}{\partial x}\right] \approx [\eta_i(\psi_{i+1} - \psi_i) - \eta_{i-1}(\psi_i - \psi_{i-1})]. \quad (2)$$

According, the discrete one-dimensional version of the wave equation can be obtained as

$$\eta_i(\psi_{i+1} - \psi_i) - \eta_{i-1}(\psi_i - \psi_{i-1}) + \omega^2\psi_i = 0. \quad (3)$$

To construct a diluted random elasticity distribution we will follow the formalism used in the 1D diluted Anderson model [36–41]. The diluted Anderson model was constructed by introducing a pure site with on-site energy ϵ_0 between each original pair of neighboring Anderson sites [36]. Therefore, to mimic this kind of short-range correlation, the elastic constants η_i will be considered as

$$\begin{aligned} \eta_i &= \eta_m + x_j \qquad \text{for} \quad i \quad \text{odd} \\ \eta_i &= \eta_m \qquad \text{for} \quad i \quad \text{even}, \end{aligned} \quad (4)$$

where $\eta_m = 2$ and x_j are uncorrelated random numbers within the interval $[-0.5, 0.5]$. The localization properties and acoustic wave propagation will be measured according standard tools used in literature [13]. The degree of localization will be obtained using the well known localization length (inverse of Lyapunov exponent $\lambda = 1/\gamma$). For extended states $\lambda/N \approx$ const while it goes to zero for localized waves. The most accurate numerical method to compute the localization length is the transfer matrix method (TMM) [7,13]. The (TMM) is obtained from a matrix recursive reformulation of equation (3). The matrix equation is

$$\begin{pmatrix}\psi_{i+1}\\ \psi_i\end{pmatrix} = \begin{pmatrix}\frac{-\omega^2+\eta_i+\eta_{i-1}}{\eta_i} & -\frac{\eta_{i-1}}{\eta_i}\\ 1 & 0\end{pmatrix}\begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix} = T_i\begin{pmatrix}\psi_i\\ \psi_{i-1}\end{pmatrix}. \quad (5)$$

The wave amplitude of the complete 1D system is given by the product of the transfer matrices $Q_N = \prod_{i=1}^{N} T_i$. The logarithm of the smallest eigenvalues of the limiting matrix $\Gamma = \lim_{N\to\infty}(Q_N^\dagger Q_N)^{1/2N}$ defines the Lyapunov exponent γ (inverse of localization length $\lambda = 1/\gamma$). Further details about the computation of this parameter can be found in [1,2,13]. Typically, we use up to $N = 2^{17}$ transfer matrices to compute the localization length. Moreover, we apply the finite-difference method with second-order discretization for both, time and spatial variables, proposed in reference [13]. Thus, in discretized form, $\psi(x,t)$ is written as ψ_i^n, where n denotes the time step number and i is the grid point number [13]. Therefore, the second time derivative in equation (1) is given by [13]

$$\frac{\partial^2}{\partial t^2}\psi(x,t) \approx \frac{\psi_i^{n+1} - 2\psi_i^n + \psi_i^{n-1}}{\Delta t^2}, \quad (6)$$

where Δt is the size of the time step. The spatial derivative will be written as

$$\frac{\partial}{\partial x}\left[\eta(x)\frac{\partial\psi(x,t)}{\partial x}\right] \approx \frac{1}{\Delta x^2} \left[\eta_i(\psi_{i+1}^n - \psi_i^n) - \eta_{i-1}(\psi_i^n - \psi_{i-1}^n)\right]. \quad (7)$$

In our calculations the spacing Δx between two neighboring grid points was kept $\Delta x = 1$. In order to ensure the stability of the discretized equations we used $\Delta t < \Delta x/100$. We carried our dynamical analysis by sending a wave from one side of the chain ($L = 0$) and recording the transmitted wave at position L. We calculated the intensity spectrum of the transmitted wave at position L defined as

$$A(\omega) = (1/2)|\psi_L(\omega)|^2 \quad (8)$$

where $\psi_L(\omega)$ is the Fourier transform of the transmitted wave $\psi_L(t)$ at position $L = 10\,000$. For transmitted acoustic modes, $A(\omega) > 0$ and goes to zero for filtered ones. In our dynamical calculations the chain length was $N = 2^{15}$.

3 Results

We first present the calculations of the typical localization length using the transfer-matrix technique for a very large chain ($N \approx 5 \times 10^6$). It should be stressed that in this method self-averaging effects automatically takes care of statistical fluctuations. The resulted data has statistical errors less than 5%. We estimate and control these statistical fluctuations following the deviations of the calculated eigenvalues of two adjacent iterations [1,2,13]. In Figure 1a (solid line) we show the scaled localization length λ/N versus ω computed for a diluted random system. In Figure 1a (dashed line), we show the results for the scaled localization length λ/N versus ω obtained from an uncorrelated random system with $\eta_i = 2 + a_i$ where a_i are uncorrelated random numbers within the interval $[-0.5, 0.5]$. All calculations were averaged over 30 disorder configurations. For

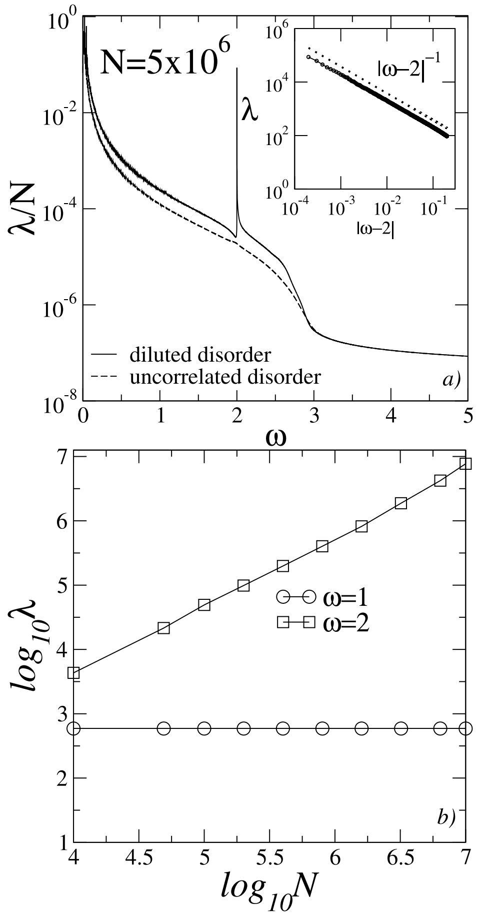

Fig. 1. (a) Scaled localization length λ/N versus ω computed for a diluted random system (solid line) and uncorrelated random system (dashed line). For the uncorrelated case, the localization length diverges only for $\omega = 0$. In the diluted case we obtain a resonant mode with $\lambda \approx N$ for $\omega = 2$. In the inset we show that λ diverges around the critical frequency as $\lambda \propto |\omega - 2|^{-1}$. (b) A quantitative scaling analysis of the localization length showing that the localization length at resonant mode diverges proportional to the system size ($\lambda(\omega = 2) \propto N^{0.95(5)}$).

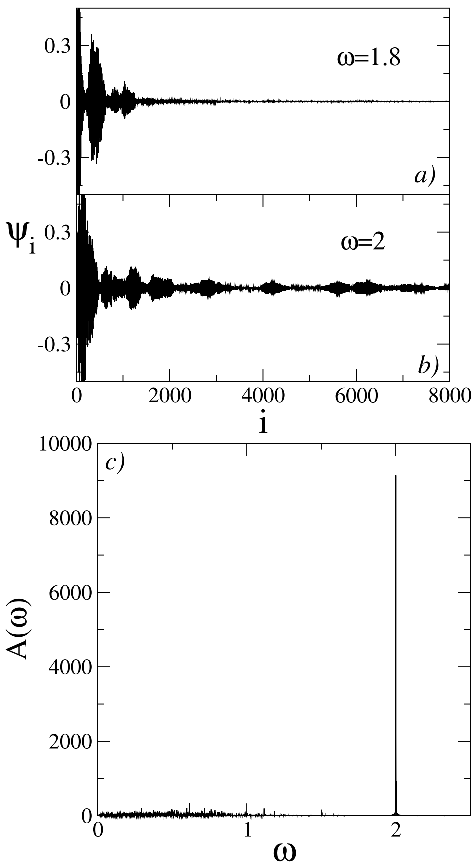

Fig. 2. (a, b) The wave amplitude ψ_i versus grid index i at time $t = 5\,000\,000\Delta t$ computed by solving the wave equation of a diluted random media. The incident wave is a sine wave with frequency (a) $\omega = 1$ and (b) $< \omega = 2$. The incident wave propagates through the diluted disordered media for $\omega = 2$ while it is damped in the case of $\omega = 1$. (c) The intensity spectrum $A(\omega)$ versus frequency for these simulations. We observe that the medium behaves as a filter to transmit only the mode at frequency $\omega = 2$.

the uncorrelated case, the localization length scales proportionally to the system size only for $\omega = 0$. However, the diluted disorder promotes the emergence of a resonant mode with $\lambda/N \approx$ const. for $\omega = \eta_m = 2$. In the inset of Figure 1a we plot the localization length around the critical frequency $\omega = 2$. We found that at the resonant mode λ diverges as $\lambda \propto |\omega - 2|^{-1}$. A quantitative scaling analysis of the localization length is showed in Figure 1b. We plot λ versus N for $\omega = 1$ and 2. All calculations were averaged over 30 disorder configurations and N was used from 10^4 up to 10^7. Within our numerical precision, the localization length at the resonant mode diverges proportional to the system size ($\lambda(\omega = 2) \propto N^{0.95(5)}$). This result suggests the possibility of an extended acoustic wave at frequency $\omega = 2$. However we stress that the divergence of the localization length itself does not guarantee the existence of extended states, as in the case of a vibrational wave envelope displaying a power-law decay [1,2]. Therefore, following the standard framework of localization theory, we must investigate the influence of the above extended states by directly solving the scalar wave equation. We follow the time propagation of an incident wave and directly obtained the degree of transmittance of a diluted random media. In Figure 2a, 2b we plot the wave amplitude ψ_i versus grid index i at time $t = 5\,000\,000\Delta t$. The incident wave was a sine wave with frequency (a) $\omega = 1$ and (b) $\omega = 2$. We observe that for $\omega = 2$ the incident wave displays a propagation through the diluted disordered media while it is damped in the case of $\omega = 1$. We also considered the incident wave as a pulse defined by $\Psi_0(t) = \exp[-(t-t_0)^2/2\sigma_t^2]\cos(\omega t)$ with $\sigma_t = (1/\sigma_\omega) = 100$ and no qualitative change in the physical properties was found. Calculations were done using $N = 30\,000$ points. In Figure 2c we present the resulting frequency dependence of the intensity spectrum $A(\omega)$ for these simulations. All the modes with $\omega \neq 2$ decay, and

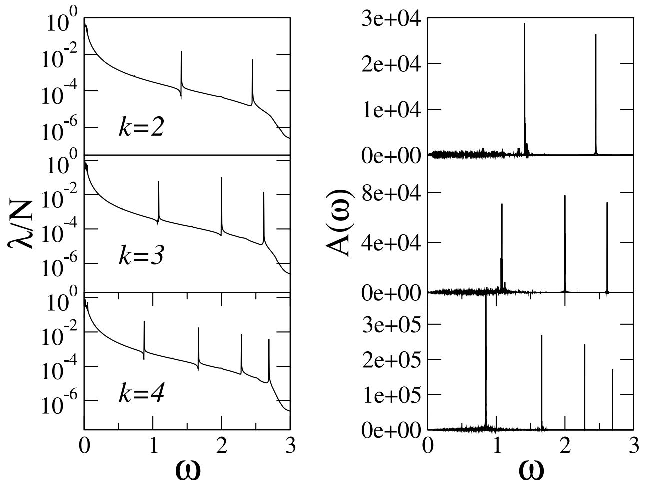

Fig. 3. Left panel: the scaled localization length λ/N versus ω computed for a diluted random system where each original pair of neighboring disordered points was diluted by introducing a sequence of k new points with elasticity $\eta_m = 2$. We use $k = 2$, 3 and 4, $N = 5 \times 10^6$ points and averaged over 30 disorder configurations. We observe a multi-peak structure where the number of resonances is equal to the number of diluting points. Right panel: the intensity spectrum $A(\omega)$ versus ω by considering the incident wave as a sine wave with frequency tuned on the resonances found in the left panel. Calculations were done by using $N = 30\,000$ points and time $t = 15\,000\,000\Delta t$. In agreement with the localization length calculations, the dynamical formalism provides the same resonance set.

the medium behaves as a filter to transmit only the modes at frequency $\omega = 2$. Before ending this section, let us call attention to another kind of diluted disorder distribution. In references [40,41], the diluted disorder was extended to include a general diluting function which defines the on-site energies within each non-random segment. Therefore, we now construct the diluted disorder by introducing a sequence of k new sites between each original pair of neighboring disordered sites. These sequences are all identical and the elasticity within such sequences are given by $\eta_l = 2$ with $l = 1, 2, \ldots, k$. In Figure 3 (left panel) we show the scaled localization length λ/N versus ω computed for an extended diluted random system with $k = 2$, 3 and 4. Calculations were done for $N = 5 \times 10^6$ points and averaged over 30 disorder configurations. We observe in Figure 3a (left panel) a multi-peak structure where the number of resonances is equal to the number of diluting points. The right panel (Fig. 3b) shows the intensity spectrum $A(\omega)$ versus ω by considering the incident wave as a sine wave with frequency tuned on the resonances found in Figure 3a (left panel). Calculations where done by using $N = 30\,000$ points and time $t = 15\,000\,000\Delta t$. The dynamical formalism is in perfect agreement with localization length calculation, leading to the the same resonance set. Therefore, our calculations indicates that the extended version of the diluted disorder previously studied in electronic models can be used to manufacture tunable filters of acoustic waves.

4 Summary, discussions and conclusions

In this paper we have contributed for further understanding of acoustic waves propagation in low-dimensional systems with correlated disorder distribution. We considered a discrete one-dimensional version of the wave equation where the elasticity distribution appears as an effective spring constant [13]. The correlated random system was assumed to be composed of two interpenetrating chains with pure and random elasticity intensities. By using a transfer-matrix method we computed the localization length of the allowed acoustic waves. Our results showed that the diluted disorder distribution used here promotes the emergence of extended acoustic waves with divergent localization length. In addition, we found that at the resonant mode λ diverges as $\lambda \propto |\omega - \omega_c|^{-1}$. Moreover, by using a dynamical method based on numerical solution of the scalar wave equation for the propagation of an acoustic wave packet, we numerically demonstrated that the chain indeed localizes all the frequencies except those tuned on the resonance frequency. Both formalisms provide an accurate estimate of the resonance frequencies. The results found here show the possibility of wave propagation in a disordered media at high-frequencies. However, the resonances associated with the diluted disorder has a specificity that drastically slows the wave dynamics: the localization length diverges linearly around the critical frequency in contrast with dimer like correlations [13] where the localization length diverges with $\lambda \propto |\omega - \omega_c|^{-2}$. We expect that the present work will stimulate further theoretical and experimental investigations along this line.

This work was partially supported by CNPq, CAPES, and FINEP (Federal Brazilian Agencies), CNPq-Rede Nanobioestruturas, as well as FAPEAL (Alagoas State Agency). The research work of A.E.B. Costa is supported by a graduate programa of CAPES. We are grateful to Professors Dr. M.L. Lyra, W.S Dias, Dr. I. Gléria, Dr. R.A. Caetano and Dr. R.P.A. Lima for useful discussions, corrections and suggestions.

References

1. B. Kramer, A. MacKinnon, Rep. Prog. Phys. **56**, 1469 (1993)
2. I.M. Lifshitz, S.A. Gredeskul, L.A. Pastur, *Introduction to the Theory of Disordered Systems* (Wiley, New York, 1988)
3. R.A. Romer, H. Schulz-Baldes, Europhys. Lett. **68**, 247 (2004)
4. E. Abrahams, P.W. Anderson, D.C. Licciardello, T.V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979)
5. F.A.B.F. de Moura, M.D. Coutinho-Filho, E.P. Raposo, M.L. Lyra, Phys. Rev. B **66**, 014418 (2002)
6. F.A.B.F. de Moura, Eur. Phys. J. B **58**, 389 (2007)
7. F.A.B.F. de Moura, M.D. Coutinho-Filho, E.P. Raposo, M.L. Lyra, Phys. Rev. B **68**, 012202 (2003)
8. F.A.B.F. de Moura, F. Domínguez-Adame, Eur. Phys. J. B **66**, 165 (2008)
9. D. Sornette, Phys. B **219**, 320 (1996)

10. S.M. Cohen, J. Machta, T.R. Kirkpatrick, C.A. Condat, Phys. Rev. Lett. **58**, 785 (1987)
11. B.C. Gupta, Z. Ye, Phys. Rev. E **67**, 036606 (2003)
12. V. Baluni, J. Willemsen, Phys. Rev. A **31**, 3358 (1985)
13. A. Esmailpour, M. Esmailpour, A. Sheikhan, M. Elahi, M.R.R. Tabar, M. Sahimi, Phys. Rev. B **78**, 134206 (2008)
14. F. Shahbazi, A. Bahraminasab, S.M.V. Allaei, M. Sahimi, M.R.R. Tabar, Phys. Rev. Lett. **94**, 165505 (2005)
15. R. Sepehrinia, A. Bahraminasab, M. Sahimi, M.R.R. Tabar, Phys. Rev. B **77**, 014203 (2008)
16. A. Bahraminasab, S.M.V. Allaei, F. Shahbazi, M. Sahimi, M.D. Niry, M.R.R. Tabar, Phys. Rev. B **75**, 064301 (2007)
17. A. Bahraminasab, A. Esmailpour, S.M.V. Allaei, F. Shahbazi, M. Sahimi, M.R.R. Tabar, Phys. Rev. B **77**, 216302 (2008)
18. S. He, J.D. Maynard, Phys. Rev. Lett. **57**, 3171 (1986)
19. J.D. Maynard, Rev. Mod. Phys. **73**, 401 (2001)
20. M. Hilke, Phys. Rev. A **80**, 063820 (2009)
21. A. Crisanti, J. Phys. A **23**, 5235 (1990)
22. E.M. Nascimento, F.A.B.F. de Moura, M.L. Lyra, Photonics Nanostruct. **7**, 101 (2009)
23. E.M. Nascimento, F.A.B.F. de Moura, M.L. Lyra, Opt. Express **16**, 6860 (2008)
24. J.C. Flores, J. Phys.: Condens. Matter **1**, 8471 (1989)
25. D.H. Dunlap, H.-L. Wu, P.W. Phillips, Phys. Rev. Lett. **65**, 88 (1990)
26. P.W. Phillips, H.-L. Wu, Science **252**, 1805 (1991)
27. A. Sánchez, F. Domínguez-Adame, J. Phys. A **27**, 3725 (1994)
28. A. Sánchez, E. Maciá, F. Domínguez-Adame, Phys. Rev. B **49**, 147 (1994)
29. E. Diez, A. Sánchez, F. Domínguez-Adame, Phys. Rev. B **50**, 14359 (1994)
30. F. Domínguez-Adame, E. Diez, A. Sánchez, Phys. Rev. B **51**, 8115 (1995)
31. V. Bellani, E. Diez, R. Hey, L. Toni, L. Tarricone, G.B. Parravicini, F. Domínguez-Adame, R. Gómez-Alcalá, Phys. Rev. Lett. **82**, 2159 (1999)
32. F.A.B.F. de Moura, M.L. Lyra, Phys. Rev. Lett. **81**, 3735 (1998)
33. F.A.B.F. de Moura, M.L. Lyra, Physica A **266**, 465 (1999)
34. F.M. Izrailev, A.A. Krokhin, Phys. Rev. Lett. **82**, 4062 (1999)
35. U. Kuhl, F.M. Izrailev, A.A. Krokhin, H.-J. Stöckmann, Appl. Phys. Lett. **77**, 633 (2000)
36. M. Hilke, J. Phys. A **30**, L367 (1997)
37. E. Lazo, M.E. Onell, Physica B **299**, 173 (2001)
38. F. Domínguez-Adame, I. Gomez, A. Avakyan, D. Sedrakyan, A. Sedrakyan, Phys. Stat. Sol. B **221**, 633 (2000)
39. W. Deng, Physica B **279**, 224 (2000)
40. E. Lazo, M.E. Onell, Phys. Lett. A **283**, 376 (2001)
41. F.A.B.F. de Moura, M.N.B. Santos, U.L. Fulco, M.L. Lyra, E. Lazo, M.E. Onell, Eur. Phys. J. B **36**, 81 (2003)
42. S.S. Albuquerque, F.A.B.F. de Moura, M.L. Lyra, E. Lazo, Phys. Lett. A **355**, 468 (2006)
43. M. Hilke, Phys. Rev. Lett. **91**, 226403 (2003)
44. Y. Hanein, U. Meirav, D. Shahar, C.C. Li, D.C. Tsui, H. Shtrikman, Phys. Rev. Lett. **80**, 1288 (1998)
45. S. Bogdanovich, D. Popovic, Phys. Rev. Lett. **88**, 236401 (2002)