

# Scaling laws for the transmission of random binary dielectric multilayered structures

E. M. Nascimento, F. A. B. F. de Moura, and M. L. Lyra

*Instituto de Física, Universidade Federal de Alagoas, 57072-970 Maceió, Alagoas, Brazil*

(Received 21 May 2007; published 20 September 2007)

We investigate several scaling aspects of the transmission spectrum of disordered one-dimensional dielectric structures. We consider a binary stratified medium composed of a random sequence of  $N$  slabs with refraction indices satisfying the Bragg condition. The mode for which the optical thickness corresponds to half wavelength is insensitive to disorder and fully transparent. The average transmission in a frequency range around this resonance decays as  $1/N^{1/2}$ , and the localization length diverges quadratically as this resonance mode is approached. In the vicinity of the quarter-wavelength mode, the localization length diverges logarithmically and the frequency averaged transmission exhibits a stretched exponential dependence on the total thickness. At the quarter-wavelength resonance, the Lyapunov exponent for different realizations of disorder has a Gaussian distribution leading to distinct scaling laws for the geometric and arithmetic averages of the transmission. The scaling laws for the half- and quarter-wavelength modes are analogous to those found in electronic one-dimensional Anderson models with random dimers and pure off-diagonal disorder, respectively, which are known to display similar violations of the usual exponential Anderson localization.

DOI: [10.1103/PhysRevB.76.115120](https://doi.org/10.1103/PhysRevB.76.115120)

PACS number(s): 78.67.Pt, 42.25.Dd, 78.20.Bh, 72.15.Rn

## I. INTRODUCTION

The study of wave transport in random media has been a subject of renewable interest during the last decades, motivated by its inherent importance to the understanding of electric, magnetic, mechanic, and optical properties of matter with potential applications to the development of new devices. Within this scenario, the Anderson theory for noninteracting electrons in random media plays a central role.<sup>1-4</sup> Although initially intended to study electronic transport in random media, the predictions of the Anderson theory have a quite broad range of applications which extend to general wave transport phenomena.

According to scaling arguments,<sup>2</sup> three-dimensional systems with weak disorder may sustain extended modes over the whole sample, although with a finite coherence length. In this regime, wave propagation can occur, thus leading to most of the transport properties of condensed matter. For strong disorder, the incoherent interference of waves scattered by impurities leads to an exponential localization and, consequently, to the absence of long distance transport. The exponential localization is predominant in low-dimensional systems. In particular, the scaling theory of Anderson localization predicts that all wave modes shall be exponentially localized for any amount of disorder in one dimension.<sup>2-4</sup>

Violations of the exponential localization in one-dimensional (1D) disordered systems have been reported in a series of model systems. The random-dimer model,<sup>5,6</sup> which consists of a random binary chain with one of the species always appearing in pairs, has a resonant mode with no backscattering due to dimers that remain extended. In the Anderson chain with diluted disorder, i.e., with disorder present just in a given sublattice, there is a Bloch state with vanishing amplitudes at the disordered sublattice that also remains delocalized.<sup>7-10</sup> The presence of resonant delocalized modes promotes a diffusivelike spread of initially localized wave packets, thus being relevant for electronic transport. Models that include long-range correlated disorder have also been

shown to display a band of extended states for strong enough correlations which can sustain coherent Bloch oscillations in the presence of a static electric field.<sup>11-13</sup> Another class of models that exhibit nonexponentially localized modes is that with chiral symmetry, such as the tight-binding 1D model with pure off-diagonal disorder and only first-neighbor couplings.<sup>14,15</sup> This model has a special mode at the band center whose wave-function envelope has an asymptotic stretched exponential tail. Another mechanism for the emergence of nonexponentially localized states in 1D random systems is through the hybridization of spatially separated degenerate modes. Recently, the existence of these so-called necklace states<sup>16</sup> has been reported by transmission measurements in random dielectric multilayers.<sup>17</sup>

Actually, the propagation of electromagnetic waves in random media corresponds to the ideal physical scenario to apply the concepts of Anderson localization, since photons are truly noninteracting particles.<sup>18</sup> Anderson localization of light waves has indeed been observed in disordered materials.<sup>19-21</sup> In 1D disordered system, the localized modes decay exponentially and, as a consequence, the ensemble average of the transmission logarithm over many realizations of the disorder usually decays linearly with the sample thickness  $L$ .<sup>16</sup> The simplest 1D optical disordered system corresponds to a sequence of thin dielectric layers with no translational order. A multilayer system has the advantage of being simple to fabricate using different procedures,<sup>22-24</sup> thus having potential applications in optoelectronics and optical communication.<sup>25</sup>

In the absence of disorder, the transmission spectra of periodic multilayered structures exhibit a characteristic range of frequencies with no propagating modes. Materials with such property are called *photonic crystals*,<sup>26</sup> in analogy with atomic crystals. The region of forbidden frequencies is called *photonic band gap* and, as in the electronic case, arises as a result of multiple interference of Bragg scatterings. However, due to the vectorial character of the electromagnetic field, in contrast to the scalar nature of the electronic wave function,

photonic crystals exhibit new features related to polarization and incidence angle dependent effects.<sup>27-31</sup>

The band gap in multilayered systems can be controlled by changing the physical characteristics and the positional distribution of the layers. Several interesting results have been reported for structures following quasiperiodic (pseudo-random) sequences such as the raising of non-Bragg gaps in a Thue-Morse multilayer<sup>32</sup> and the self-similarity of the energy spectrum of Fibonacci structures.<sup>33,34</sup> Scale invariant gaps have also been obtained with the utilization of metamaterials in the multilayer composition.<sup>35,36</sup> For truly random sequences, the combined effect of Bragg reflection and light localization has been explored to demonstrate a band-gap extension effect with possible applications in the design of broadband high reflectors composed of a periodic layer sequence with fluctuating optical lengths.<sup>37,38</sup> Recently, the transmission spectrum of binary multilayer structure with positional disorder, composed of dielectric slabs with the same optical length, has been reported<sup>17,39,40</sup> and shown to display peaks associated with necklace states.<sup>16</sup> Numerical results on finite-size samples showed a transmission peak when the optical length corresponds to a quarter wavelength as a signature of hidden partial order.<sup>40</sup>

In this paper, we will report the scaling behavior associated with several aspects of the transmission spectrum of a binary multilayer structure with positional disorder. We will particularly address the case on which the dielectric layers have distinct refractive indices but their thicknesses are chosen to give them the same optical length. By employing a transfer matrix calculation in finite-size samples, we will compute the transmission coefficient as a function of the mode frequency. We will drive special attention to the spectral ranges at the vicinity of the half- and quarter-wavelength modes. The transmission peaks at these frequencies originate from distinct mechanisms reflected on different scaling laws for the size dependence of the average transmission and for the localization length singularities. These scaling laws will be shown to be analogous to the ones observed in the tight-binding models for electronic states in random-dimer and random-hopping chains.

## II. TRANSFER MATRIX FORMALISM

The transfer matrix formalism is particularly suited to compute the transmission spectrum of electromagnetic waves in stratified dielectric media.<sup>41,42</sup> Here, we will sketch the main lines of the transfer matrix technique for the particular case of normal incidence. We will assume a plane wave of frequency  $\omega$  propagating along the  $z$  axis direction, which is normal to the interfaces of a dielectric slab of thickness  $d$  and linearly polarized in such a way that the electric field amplitude can be written as  $\vec{E}(z) = E(z)\hat{x}$ . The extension for the case of oblique incidence of transverse electric and transverse magnetic waves is straightforward. The relation between the electric and magnetic fields at the interface located at  $z = z_1$  and the fields at the interface at  $z = z_1 + d$  can be expressed in transfer matrix form as

$$\begin{pmatrix} E_1 \\ B_1 \end{pmatrix} = M \begin{pmatrix} E_2 \\ B_2 \end{pmatrix} = \begin{pmatrix} \cos \delta & \frac{i}{p} \sin \delta \\ ip \sin \delta & \cos \delta \end{pmatrix} \begin{pmatrix} E_2 \\ B_2 \end{pmatrix}, \quad (1)$$

where the phase change  $\delta = \omega nd/c$ ,  $n$  is the refractive index of the medium, and  $p = \sqrt{\frac{\epsilon}{\mu}}$ , where  $\epsilon$  and  $\mu$  are the dielectric constant and magnetic permeability, respectively. The boundary conditions across an interface requires the continuity of the parallel components of the fields. As a consequence, for a stratified medium consisting of a sequence of  $N$  dielectric layers, the fields at the first and last interfaces can be related through a product of individual transfer matrices as

$$\begin{pmatrix} E_0 \\ B_0 \end{pmatrix} = M_1 M_2 \cdots M_N \begin{pmatrix} E_N \\ B_N \end{pmatrix} = M \begin{pmatrix} E_N \\ B_N \end{pmatrix}, \quad (2)$$

where  $M_i$  is the transfer matrix of the  $i$ th layer,  $E_0$  and  $B_0$  are the electric and magnetic field amplitudes at the first interface, and  $E_N$  and  $B_N$  the field amplitudes at the last interface. The complex transmission coefficient of such stratified medium can be obtained by assuming that the incident beam is coming from the left and that one has just the outgoing transmitted wave at the right of the multilayer structure. It can be expressed as

$$t(\omega) = \frac{2p_i}{(m_{11} + m_{12}p_o)p_i + (m_{21} + m_{22}p_o)}, \quad (3)$$

where  $m_{ij}$ 's represent the elements of the total transfer matrix  $M$ .  $p_i$  and  $p_o$  are related to the input and output media. The complex transmission coefficient brings information regarding both the phase and amplitude of the transmitted wave. From the phase, one can have access to the dispersion properties of the wave propagating through the multilayered structure such as its group velocity. In what follows, we will be particularly interested in analyzing the ratio between the intensities of the outgoing and incoming waves, which is given by the transmission  $T(\omega) = \frac{p_o}{p_i} |t(\omega)|^2$ .

## III. TRANSMISSION SPECTRUM OF BINARY RANDOM SEQUENCES

We will consider a random stratified binary medium composed of  $N$  nonabsorbing dispersionless dielectric layers. As representative refraction indices, we will consider  $n_A = 1.45$  and  $n_B = 2.5$ , although the qualitative aspects we are going to explore remains the same for any pair  $(n_a, n_b)$ . The layers can be considered as consisting of porous silicon, whose refractive index can be made to vary in a wide range by controlling the porosity.<sup>43</sup> The  $i$ th layer of the sequence has the same probability of being type  $A$  or  $B$ . The resulting dielectric structure is surrounded by vacuum, and the layer thicknesses will be taken in such a way as to satisfy the Bragg condition, i.e., both kinds of dielectric layers will have the same optical length  $n_A d_A = n_B d_B = \lambda_0$ . In what follows, the characteristic frequency  $\nu_0 = c/\lambda_0$  corresponds to the mode whose wavelength in vacuum equals the layer's optical length.

In order to have an overall picture of the role played by disorder, we compared the transmission spectrum of a peri-

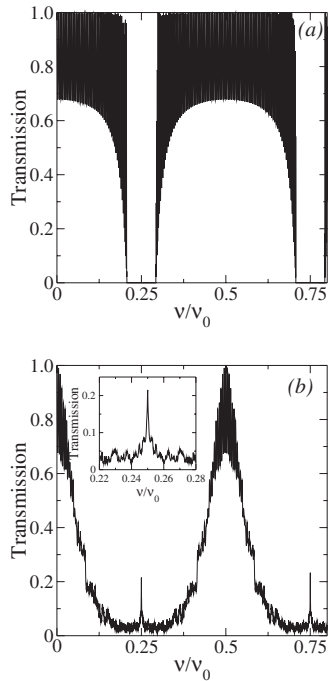


FIG. 1. Transmission spectra of (a) periodic and (b) random stratified binary media with  $N=10^2$  layers. For the random structure, we performed an average over  $10^2$  distinct realizations of disorder, which plays quite distinct roles in the range of frequencies corresponding to transmitting and nontransmitting bands. The inset shows in detail the disorder induced transmission peak at the center of the stop band.

odic sequence of alternating  $A$  and  $B$  layers with that of a random multilayer sequence, as shown in Fig. 1 for structures with  $N=10^2$  layers. The periodic sequence constitutes the so-called distributed Bragg reflector. The transmission spectrum displays a sequence of transmitting and nontransmitting bands. The transmitting bands are centered at frequencies for which the optical length of each layer is an integer multiple of the half wavelength in vacuum, i.e., the phase change in each layer is  $\delta=m\pi$ , with  $m$  integer. The multilayer structure is fully transparent to these modes. The nontransmitting photonic band gaps are centered at frequencies that match the optical length to be a quarter wavelength displaced from the multiples of half wavelengths [phase change  $\delta=(m+1/2)\pi$ ]. The width of the photonic band gap depends on the relation between the refraction indices  $n_a$  and  $n_b$ .

The transmission spectrum of random sequences, averaged over  $10^2$  realizations of the disorder, is depicted in Fig. 1(b). Disorder has quite distinct effects in the range of frequencies corresponding to transmitting and nontransmitting bands. Firstly, one notices that the high transmission at the center of the transmitting bands is not affected by disorder, as expected once the transfer matrix of each layer becomes an identity matrix  $I$  for these modes (actually  $\pm I$ ). However, the width of the transmission band is much narrower. Within the nontransmitting band, disorder induces the emergence of a few modes. These are related to necklace states which result from hybridization of degenerate states localized in dis-

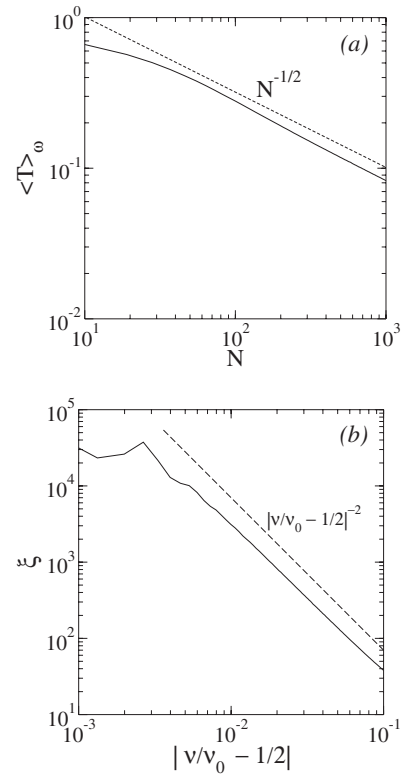


FIG. 2. (a) Spectral average of the transmission versus the number of layers  $N$ . (b) Localization length in the vicinity of the half-wavelength mode. The average transmission decays with  $1/N^{1/2}$ , and the localization length diverges quadratically as one approaches the resonance mode. The estimate of the localization length was performed considering a finite structure with  $N=10^4$  layers, and the averages were taken over  $10^3$  distinct random sequences. The saturation of the localization length very close to the resonance frequency is a finite-size effect.

tinct regions of the structure.<sup>17,40</sup> Although these modes become rare as the number of layers increases, they dominate the average transmission in this frequency range. Further, the average transmission spectrum develops a narrow peak at the center of the photonic band gap.

The insensitivity of the fully transmitting mode on the spatial arrangement of the layers resembles the violation of the Anderson localization in the random-dimer model.<sup>5,6</sup> This model also has a resonant mode for which the dimers become transparent. A typical signature of this kind of resonance is that the localization length  $\xi$  shall diverge as  $|E - E_0|^{-2}$ . Consequently, the energy range of effectively extended states with  $\xi > N$  shall decrease as  $N^{-1/2}$ . We test for the above scalings on the vicinity of the half-wavelength mode. As a measure of the localization length  $\xi$ , we considered the inverse of the Lyapunov exponent [ $\xi = 1/\Lambda = -\lim_{N \rightarrow \infty} (N/\ln T)$ ]. We also computed the spectral average of the transmission for the frequency range corresponding to the transmission band of the periodic sequence. Its size dependence shall reflect the narrowing of the transmission peak. In Fig. 2, we report our results for these quantities. As anticipated above, the average transmission decays as  $N^{-1/2}$ , and the localization length diverges quadratically as one approaches this resonance mode.

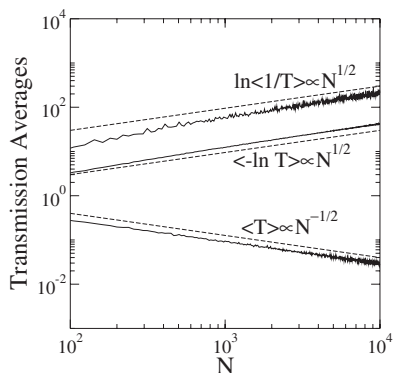


FIG. 3. Size dependence of the geometric ( $\exp\langle\ln T\rangle$ ), harmonic ( $\langle(1/T)^{-1}\rangle$ ), and arithmetic ( $\langle T\rangle$ ) average values of the transmission at the quarter-wavelength resonance frequency. Both the geometric and harmonic averages display a stretched exponential scaling, while the arithmetic average exhibits a slower power-law decay. Here, we averaged over  $10^4$  distinct random sequences.

At the center of the photonic band gap of the periodic sequence, disorder promotes a narrow peak of the transmission. At this frequency, one can notice that the transfer matrices of both kinds of layer become off diagonal, i.e., have null diagonal elements. However, the off-diagonal elements will be randomly distributed. This scenario resembles the one achieved for one-dimensional electronic tight-binding Hamiltonians with only random hoppings.<sup>14,15</sup> The state at the middle of the energy band of this model has an infinite localization length. In spite of this, the disorder averaged transmission approaches zero as the size of the system increases. On the basis of the central limit theorem, it has been argued that the logarithm of the transmission at this resonance shall have a Gaussian distribution in the regime of large  $N$ , whose mean square deviation grows as  $N^{1/2}$  as shown in Ref. 14. Following this distribution, the geometric [ $\exp(\langle\ln(T)\rangle)$ ] and harmonic ( $\langle(1/T)^{-1}\rangle$ ) mean values of the transmission shall behave as  $\exp(-\alpha\sqrt{L})$ , while the arithmetic mean value ( $\langle T\rangle$ ) shall follow a power law  $N^{-1/2}$ . As one approaches the band center, the localization diverges logarithmically,<sup>15</sup> which implies that the energy range of effectively extended states shall be very narrow, scaling as a stretched exponential.

We have verified the above scaling behavior at the quarter-wavelength mode. In Fig. 3, we report the size dependence of the geometric, harmonic, and arithmetic average values of the transmittance. Both the geometric and harmonic averages indeed display a stretched exponential scaling, while the arithmetic average exhibits a slower power-law decay. In Fig. 4, we display the probability distribution function of the transmittance logarithm at the quarter-wavelength condition as obtained from  $10^4$  distinct realizations of the disorder in structures with  $10^4$  layers. The numerically obtained distribution is well fitted by a Gaussian, thus corroborating the central limit theorem prediction. It is worth mentioning that disorder configurations that lead to high transmittance occur with larger probability than those leading to low transmittance. This is a somewhat counterintuitive aspect once disorder is expected to favor localization.

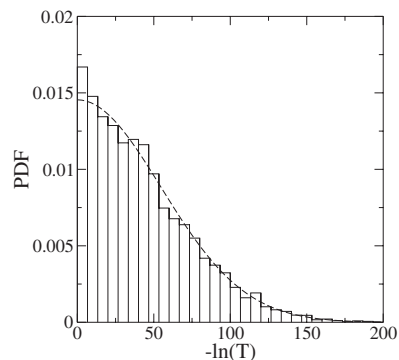


FIG. 4. Probability distribution function of the transmission logarithm at the quarter-wavelength resonance frequency. The numerically obtained distribution is well fitted by a Gaussian (dashed line), thus corroborating the central limit theorem prediction. Data were the same as those used in Fig. 3 for  $N=10^4$  layers.

Actually, at this frequency, the system has a hidden partial order. Pairs of neighboring layers of the same kind are transparent. Therefore, the effective size of the system can be renormalized by decimating such pairs until a level at which the remaining layers form a periodic alternate sequence.<sup>40</sup> The probability that such decimation proceeds up to a high order is large, thus resulting in a maximum of the probability distribution function for high transmissions.

The spectral average of the transmittance in a frequency range around the quarter-wavelength mode is shown in Fig. 5(a). In contrast with the power-law scaling obtained at the vicinity of the half-wavelength mode, we now find that the transmission spectral average exhibits a faster stretched exponential decay as the number of layers is increased. This trend is consistent with the slow logarithmic divergence of the localization length when approaching this resonant mode, as shown in Fig. 5(b). All the reported scaling behaviors at the quarter-wavelength resonance are consistent with those of the electronic tight-binding random-hopping model and shall hold for general one-dimensional models with pure off-diagonal disorder.

The above analysis showed that disorder plays opposite trends in the transmission behavior near half- and quarter-wavelength modes. Near the half-wavelength condition, i.e., at the transmission band of the periodic sequence, the main effect of disorder is to promote the exponential localization of the modes, except at the resonance. Therefore, the transmission spectral average is reduced by disorder. On the other hand, within the photonic band gap, disorder promotes the emergence of states that support a small but finite transmission in finite systems.

In order to explicitly show these opposite trends, we computed the spectral average of the transmission around the half- and quarter-wavelength resonances as a function of the disorder strength. We started with a periodic sequence of alternate  $A$  and  $B$  layers. Then, each layer of this sequence is replaced by a layer of the other species with probability  $q$ . For  $q=0$ , one retains the fully periodic sequence, while the  $q=1/2$  limit recovers the uncorrelated fully disordered sequence. Our results are shown in Fig. 6 for  $N=10^2$  layers, averaged over  $5 \times 10^3$  disorder configurations and within

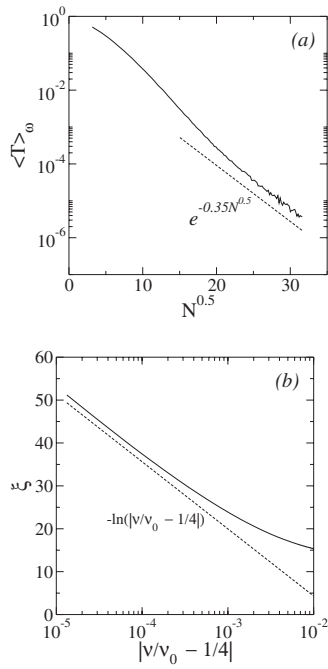


FIG. 5. (a) Spectral average of the transmittance in a frequency range around the quarter-wavelength mode as a function of the number of layers. It exhibits an asymptotic stretched exponential decay as the number of layers is increased. (b) The disorder averaged localization length in the vicinity of the quarter-wavelength resonance showing its slow logarithmic divergence. Here, we averaged over  $10^4$  random sequences. The localization length in (b) was estimated from structures having  $5 \times 10^3$  layers.

spectral ranges corresponding to the transmission (a) and reflecting (b) bands. Notice that the average transmission around the half-wavelength mode decays with increasing disorder strengths once Anderson localization is the predominant effect. The spectrally averaged transmission around the quarter-wavelength resonance depicts the opposite trend, growing as the disorder strength is increased due to the emergence of states within the gap. The average transmission is small due to the low probability of occurrence of extended necklace states.

#### IV. SUMMARY AND CONCLUSIONS

In summary, we investigated the scaling behavior of the transmission spectrum in stratified dielectric media composed of binary random sequences of  $N$  layers satisfying the Bragg reflection condition. We paid special attention to the resonance states for which the optical length of the layers corresponds to half and quarter of the mode wavelength.

The half-wavelength resonance is at the center of the transmission band of the corresponding periodic sequence of alternating layers. It is fully transparent irrespective to disorder. We numerically demonstrated that the transmission bandwidth is a decreasing function of the total number of layers and disorder strength, with the spectrally averaged transmission scaling as  $N^{-1/2}$ . Further, the localization length diverges quadratically as one approaches the resonance fre-

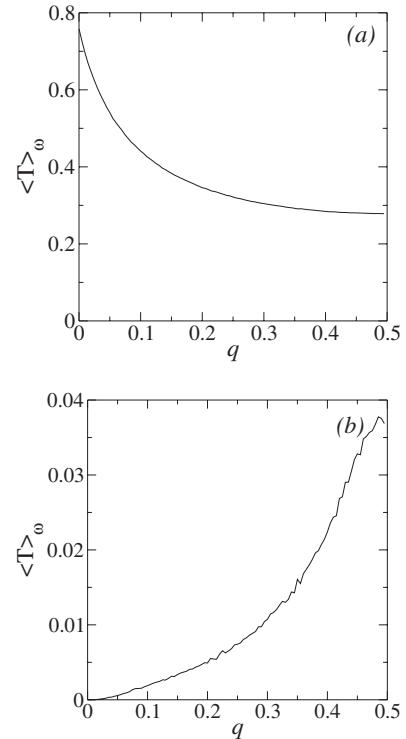


FIG. 6. Spectral average of the transmission around (a) the half- and (b) the quarter-wavelength resonances as a function of the disorder strength. Disorder plays opposite trends in the transmission behavior near each resonance. Around the half-wavelength resonance, Anderson localization is predominant and the transmission decreases with increasing disorder. On the other hand, near the quarter-wavelength resonance, disorder induces the emergence of states within the stop band and thus promotes a small transmittance in this frequency range. Data were obtained from  $5 \times 10^3$  random sequences with  $10^2$  layers each.

quency. These scaling laws are the same ones appearing in the random-dimer tight-binding Hamiltonian model for one-electron states.

At the quarter-wavelength mode, the logarithm of the transmission has a Gaussian distribution when considering distinct disorder configurations whose width scales as  $N^{1/2}$  in the limit of large  $N$ . This Gaussian distribution reflects a hidden partial order which can be revealed by decimating the pairs of neighboring similar layers. At this resonance, the geometric and harmonic averages of the transmission scale as stretched exponentials of the total number of layers, while the arithmetic average displays a slower power-law decay proportional to  $N^{-1/2}$ . Further, the spectral average of the transmission around the quarter-wavelength resonance also decays with the total thickness as an stretched exponential, which is consistent with a slow logarithmic divergence of the localization length. However, the transmission within this frequency range increases with the disorder strength. The scaling laws at this resonance are similar to the ones present in the Hamiltonian model for one-electron states in chains with random hoppings and might hold for general 1D systems with pure off-diagonal disorder.

In summary, a random binary dielectric multilayer satisfying the Bragg reflection condition exhibits two of the

known scenarios that lead to violations of the Anderson exponential localization due to disorder. Once the propagation of electromagnetic waves in random media represents the ideal setup for testing the predictions associated with the Anderson theory of localization, it would be interesting to have experimental observations of the herein reported scaling laws. However, fluctuations in the layer thicknesses of real systems shall be finely controlled to allow the Bragg condition to be closely satisfied. These fluctuations may suppress the resonance transmission peaks of thick multilayer

structures, especially the narrow peak corresponding to the quarter-wavelength condition.

#### ACKNOWLEDGMENTS

We would like to acknowledge the partial financial support from CAPES, CNPq, CNPq-Rede Nanobioestruturas, and FINEP (Brazilian Research Agencies), as well as from FAPEAL (Alagoas State Research Agency).

- 
- <sup>1</sup>P. W. Anderson, *Phys. Rev.* **109**, 1492 (1958).  
<sup>2</sup>E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, *Phys. Rev. Lett.* **42**, 673 (1979).  
<sup>3</sup>P. A. Lee and T. V. Ramakrishnan, *Rev. Mod. Phys.* **57**, 287 (1985).  
<sup>4</sup>B. Kramer and A. MacKinnon, *Rep. Prog. Phys.* **56**, 1469 (1993).  
<sup>5</sup>D. H. Dunlap, H.-L. Wu, and P. W. Phillips, *Phys. Rev. Lett.* **65**, 88 (1990).  
<sup>6</sup>P. W. Phillips and H.-L. Wu, *Science* **252**, 1805 (1991).  
<sup>7</sup>M. Hilke, *J. Phys. A* **30**, L367 (1997).  
<sup>8</sup>E. Lazo and M. E. Onell, *Physica B* **299**, 173 (2001); *Phys. Lett. A* **283**, 376 (2001).  
<sup>9</sup>F. A. B. F. de Moura, M. N. B. Santos, U. L. Fulco, M. L. Lyra, E. Lazo, and M. E. Onell, *Eur. Phys. J. B* **36**, 81 (2003).  
<sup>10</sup>M. Hilke, *Phys. Rev. Lett.* **91**, 226403 (2003).  
<sup>11</sup>F. A. B. F. de Moura and M. L. Lyra, *Phys. Rev. Lett.* **81**, 3735 (1998); **84**, 199 (2000).  
<sup>12</sup>F. A. B. F. de Moura and M. L. Lyra, *Physica A* **266**, 465 (1999).  
<sup>13</sup>F. Domínguez-Adame, V. A. Malyshev, F. A. B. F. de Moura, and M. L. Lyra, *Phys. Rev. Lett.* **91**, 197402 (2003).  
<sup>14</sup>G. Theodorou and M. Cohen, *Phys. Rev. B* **13**, 4597 (1976).  
<sup>15</sup>L. Fleishman and D. C. Licciardello, *J. Phys. C* **10**, L125 (1977).  
<sup>16</sup>J. B. Pendry, *Adv. Phys.* **43**, 461 (1994).  
<sup>17</sup>J. Bertolotti, S. Gottardo, D. S. Wiersma, M. Ghulinyan, and L. Pavesi, *Phys. Rev. Lett.* **94**, 113903 (2005).  
<sup>18</sup>S. John, *Phys. Rev. Lett.* **53**, 2169 (1984); P. W. Anderson, *Philos. Mag. B* **52**, 505 (1985).  
<sup>19</sup>D. S. Wiersma, P. Bartolini, A. Lagendijk, and R. Righini, *Nature (London)* **390**, 671 (1997).  
<sup>20</sup>M. Storzer, P. Gross, C. M. Aegerter, and G. Maret, *Phys. Rev. Lett.* **96**, 063904 (2006).  
<sup>21</sup>T. Schwartz, G. Bartal, S. Fishman, and M. Segev, *Nature (London)* **446**, 52 (2007).  
<sup>22</sup>R. P. Stanley, R. Houdre, U. Oesterle, M. Ilegems, and C. Weisbuch, *Appl. Phys. Lett.* **65**, 2093 (1994).  
<sup>23</sup>L. dal Negro, M. Stolfi, Y. Yi, J. Michel, X. Duan, L. C. Kimerling, J. LeBlanc, and J. Haavisto, *Appl. Phys. Lett.* **84**, 5186 (2004).  
<sup>24</sup>L. Dal Negro, C. J. Oton, Z. Gaburro, L. Pavesi, P. Johnson, A. Lagendijk, R. Righini, M. Colocci, and D. S. Wiersma, *Phys. Rev. Lett.* **90**, 055501 (2003); M. Ghulinyan, C. J. Oton, G. Bonetti, Z. Gaburro, and L. Pavesi, *J. Appl. Phys.* **93**, 9724 (2003).  
<sup>25</sup>J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals* (Princeton University Press, Princeton, 1995); C. M. Soukoulis, *Photonic Band Gaps Localization* (Plenum, New York, 1993).  
<sup>26</sup>E. Yablonovitch, *Phys. Rev. Lett.* **58**, 2059 (1987); S. John, *ibid.* **58**, 2486 (1987).  
<sup>27</sup>S. H. Fan, P. R. Villeneuve, and J. D. Joannopoulos, *Phys. Rev. B* **54**, 11245 (1996).  
<sup>28</sup>J. F. Galisteo-Lopez, F. Lopez-Tejiera, S. Rubio, C. Lopez, and J. Sanchez-Dehesa, *Appl. Phys. Lett.* **82**, 4068 (2003).  
<sup>29</sup>L. G. Wang, H. Chen, and S. Y. Zhu, *Phys. Rev. B* **70**, 245102 (2004).  
<sup>30</sup>V. Kimberg, F. Gel'mukhanov, H. Agren, E. Pent, A. Plekhanov, I. Kuchin, M. Rodionov, and V. Shelkownikov, *J. Opt. A, Pure Appl. Opt.* **6**, 991 (2004).  
<sup>31</sup>E. Pavarini, L. C. Andreani, C. Soci, M. Galli, F. Marabelli, and D. Comoretto, *Phys. Rev. B* **72**, 045102 (2005).  
<sup>32</sup>X. Jiang, Y. Zhang, S. Feng, K. C. Huang, Y. Yi, and J. D. Joannopoulos, *Appl. Phys. Lett.* **86**, 201110 (2005).  
<sup>33</sup>G. Gumbs and M. K. Ali, *Phys. Rev. Lett.* **60**, 1081 (1988).  
<sup>34</sup>W. Gellermann, M. Kohmoto, B. Sutherland, and P. C. Taylor, *Phys. Rev. Lett.* **72**, 633 (1994).  
<sup>35</sup>J. Li, L. Zhou, C. T. Chan, and P. Sheng, *Phys. Rev. Lett.* **90**, 083901 (2003).  
<sup>36</sup>H. Jiang, H. Chen, H. Li, Y. Zhang, J. Zi, and S. Zhu, *Phys. Rev. E* **69**, 066607 (2004).  
<sup>37</sup>H. Q. Li, H. Chen, and X. J. Qiu, *Physica B* **279**, 164 (2000).  
<sup>38</sup>V. A. Tolmachev, T. S. Perova, J. A. Pilyugin, and R. A. Moore, *Opt. Commun.* **259**, 104 (2006).  
<sup>39</sup>P. Sebbah, B. Hu, J. M. Klosner, and A. Z. Genack, *Phys. Rev. Lett.* **96**, 183902 (2006).  
<sup>40</sup>J. Bertolotti, M. Galli, R. Sapienza, M. Ghulinyan, S. Gottardo, L. C. Andreani, L. Pavesi, and D. S. Wiersma, *Phys. Rev. E* **74**, 035602(R) (2006).  
<sup>41</sup>F. L. Pedrotti and L. S. Pedrotti, *Introduction to Optics* (Prentice-Hall, Englewood Cliffs, NJ, 1987).  
<sup>42</sup>L. Carretero, M. Perez-Molina, P. Acebal, S. Blaya, and A. Fimia, *Opt. Express* **14**, 11385 (2006).  
<sup>43</sup>O. Bisi, S. Ossicini, and L. Pavesi, *Surf. Sci. Rep.* **38**, 5 (2000).