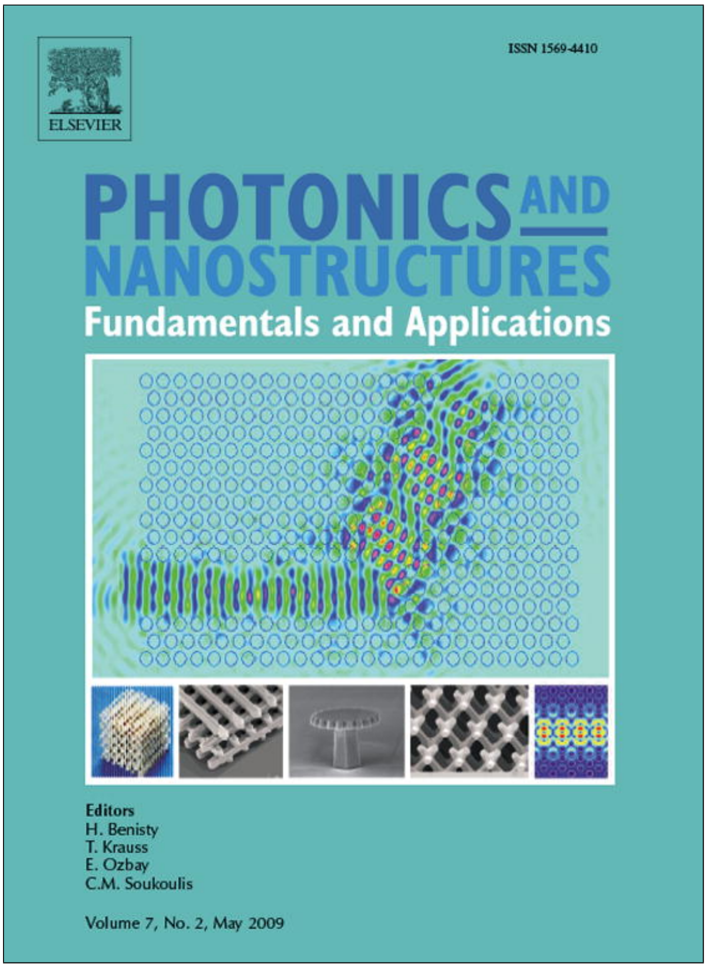
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Suppressed transmission in aperiodically modulated multilayered dielectric structures

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Abstract

We investigate the transmission of electromagnetic plane waves through 1D binary dielectric multilayered structures that exhibit aperiodic incommensurated sequences of refractive indices. The aperiodicity is introduced by considering the sequence of refractive indices to follow a sinusoidal function whose phase ϕ varies as a power-law of the layer index, $\phi \propto i^{\nu}$. For $\nu > 1$, the resulting sequence is effectively uncorrelated leading to the Anderson localization of most of the electromagnetic modes, except at the Bragg resonances. The crossover from a uniform structure at $\nu = 0$ to a quasi-periodic structure at $\nu = 1$ is signaled by a minimum at the spectrally averaged transmission. We perform a spectral analysis of the refractive index sequence to show its close connection to the main features exhibited by the averaged optical transmittance. Our results suggest that aperiodically modulated dielectric structures can potentially be used in the development of wide-band filters.

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Keywords: Multilayered structures; Transmission spectrum; Aperiodic sequences; Anderson localization

1. Introduction

The pioneer Anderson's theory concerning the localization of non-interacting electrons in disordered systems [1] led to a deeper understanding of several important condensed matter phenomena. According to the Anderson theory, the interference between multiply scattered Schroedinger waves leads to exponentially localized electron eigenfunctions in one-dimensional disordered systems. Along the last two decades, such exponential localization has been shown to be violated in a series of model systems. The random-dimer model [2,3], which consists of a random binary chain with one of the species always appearing in pairs, has a resonant mode with no back-scattering due to dimers that remain extended. Models that include long-range correlated disorders have also been shown to display a band of extended states for strongly enough correlations with can sustain coherent Bloch oscillation in the presence of a static electric field [4–6]. Another class of 1D models that can exhibit an Anderson-like localization-delocalization transition, involves non-random deterministic potentials which are incommensurate with the underlying lattice [7]. This class of models depicts features that are in between those of the Anderson model and the periodic Bloch model. The localized or extended nature of the energy eigenstates presented by this class of models has been extensively investigated in the physics literature [8,9] and has been related to the general characteristics of the aperiodic on-site energy distribution.

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As a purely interference phenomenon, Anderson localization is also observed to occur with vibrational [10–14], magnetic [15–19] and electromagnetic [20–23] waves. In fact, it has been observed in random dielectric multilayer system, where the localized modes decay exponentially and, as a consequence, the ensemble average of the transmission logarithm over many realizations of the disorder usually decays linearly with the sample thickness L.

As it occurs for the electron localization problem, violations of the exponential localization of electromagnet waves propagating in random media have been reported in several physical situations. In one-dimensional random n-mer dielectric systems, where segments with n atoms are randomly inserted in a host chain, a localization–delocalization transition of electromagnetic modes takes place at some resonance frequencies due the positional correlations in the structure [24]. Microwave-guides with spatially correlated random scatters have also been shown to support transmitting bands due to non-localized modes [25]. Metamaterials have also been extensively studied as an active component in random dielectric multilayered systems [26–28]. This kind of structure exhibits scale invariant bandgaps [26]. For a 1D disordered stacks of alternating right- and left-handed layers, the introduction of metamaterials substantially suppresses Anderson localization [27]. Structures following quasi-periodic sequences can present different kinds of bandgaps as non-Bragg gaps for Thue-Morse multilayer [29] and self-similar energy spectrum in Fibonacci-like structures [30–32]. Another mechanism for the emergence of non-exponentially localized states in 1D random systems is through the hybridization of spatially separated degenerate modes [33]. Recently, the existence of these so-called necklace states has been reported by transmission measurements in random dielectric multilayers [34,35]. Necklace states are in the origin of well-defined oscillations on the fluctuations of the transmittance in one-dimensional random photonic system with resonant layers [36].

In this work, we investigate the transmission properties of electromagnetic waves propagating through 1D binary dielectric multilayered structures that follow an aperiodic and incommensurate modulation over the refractive index sequence. In order to produce an aperiodic multilayered structure, the formalism used in Ref. [7] was considered. It consists in using a sinusoidal modulation of the refractive index sequence whose phase ϕ varies as a power-law, $\phi \propto i^{\nu}$, where i is the layer index. The exponent ν controls the degree of aperiodicity in the structure. Using a transfer matrix formalism, we compute the transmission spectrum of electromagnetic waves with frequency ω propagating along the stratified dielectric media. We will report a non-trivial transmission spectrum that strongly depends on the aperiodicity degree. In general, the spectral transmission, averaged over the frequency band, shows a maximum at $\nu = 1.0$ and a local minimum between $\nu = 0.5$ and 1.0. By employing a Fourier analysis of the refractive index distribution, we will associate the main features of the spectrally averaged transmission with the correlations present in the underlying sequence of dielectric layers.

2. Transfer matrix formalism

The transfer matrix formalism is well suited to compute the transmission spectrum of electromagnetic waves in stratified dielectric media. We are going to describe the main lines of this formalism to the particular case of normally incident plane waves of frequency ω. The stacking direction is along the z axis and the electric field linearly polarized $\vec{E}(z) = E(z)e^{-ikz}\hat{x}$, where k is the wavenumber. The relation between the amplitudes of the electric and magnetic fields at the interface located at z_1 and the fields at the interface at $z_2 = z_1 + d$, where d is the thickness of the dielectric slab, can be expressed in transfer matrix form as

$$\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = \begin{pmatrix} \cos\delta & iZ\sin\delta \\ \frac{i}{Z}\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} E_2 \\ H_2 \end{pmatrix} \quad (1)$$

where the phase change $\delta = \omega n d / c$, n is the refractive index of the medium and $Z = \sqrt{(\mu/\epsilon)}$ is the layer impedance, with ϵ and μ are the dielectric constant and magnetic permeability, respectively. The boundary conditions across an interface require the continuity of the parallel components of the electric and magnetic fields. The generalization of the above procedure for a stratified medium consisting of a sequence of N dielectric layers is straightforward. The fields at the first and last interfaces can be related through a product of individual transfer matrices as:

$$\begin{pmatrix} E_0 \\ H_0 \end{pmatrix} = M_1 M_2 \ldots M_N \begin{pmatrix} E_N \\ H_N \end{pmatrix} = M \begin{pmatrix} E_N \\ H_N \end{pmatrix} \quad (2)$$

where M_i is the transfer matrix of the ith layer, E_0 and H_0 are the electric and magnetic field amplitudes at the first interface, and E_N and H_N are the field amplitudes at the last interface. The complex transmission coefficient of such stratified medium can be obtained by assuming that the incident beam is coming from the left ant that one has just the outgoing transmitted wave at the right of the multilayer structure. It can be expressed as

$$t(\omega) = \frac{2/Z_i}{(m_{11} + m_{12}/Z_o)/Z_i + (m_{21} + m_{22}/Z_o)} \quad (3)$$

where m_{ij}'s represents the elements of the total transfer matrix M. Z_i and Z_o are related to the impedances of the input and output media. The complex transmission coefficient brings information regarding both the phase and amplitude of the transmitted wave. The phase is related to dispersion properties such as the group velocity. In what follows, we will be particularly interested in analyzing the ratio between the intensities of the outgoing and incoming waves, which is given by the transmission $T(\omega) = (Z_i/Z_o)|t(\omega)|^2$.

3. Model and results

We will consider multilayer structures composed of two kinds of non-absorbing dispersionless dielectric layers with thicknesses taken in such a way that $n_A d_A = n_B d_B$. In order to produce an aperiodic multilayered structure, we closely followed the procedure described in ref. [7]. The sequence of refractive indices is taken to obey the aperiodic rule:

$$V_j = \cos(\alpha\pi j^{\nu}) \rightarrow n_j = \begin{cases} n_A & \text{if } V_j \leq 0 \\ n_B & \text{if } V_j > 0 \end{cases} \quad (4)$$

The above rule actually uses a sinusoidal function whose phase ϕ varies as a power-law, $\phi \propto j^{\nu}$, where j labels the layers of the stratified dielectric structure. The exponent ν controls the degree of aperiodicity in the structure. The resulting dielectric structure is surrounded by vacuum. For $\nu = 1$ and rational α one has a purely periodic refractive index sequence and all electromagnetic modes shall become extended. For $\nu = 1$ and irrational α the sequence becomes quasi-periodic (incommensurate) and the spectrum of propagating modes becomes fractal [30,31]. For $\nu < 1$ the sequence is aperiodic with the typical wavelength of the refractive index modulation increasing as a function of the layer index. In this regime, the spectrum exhibits localized and delocalized modes. In the opposite regime of $\nu > 1$, the modulation wavelength decreases with the layer index and most states become localized except at specific resonances, characterizing a pseudo-random regime.

In our calculations we used $\alpha = (\sqrt{5} - 1)/2$ (the golden mean), $n_A = 1.5$ and $n_B = 2.5$. Fig. 1 shows the

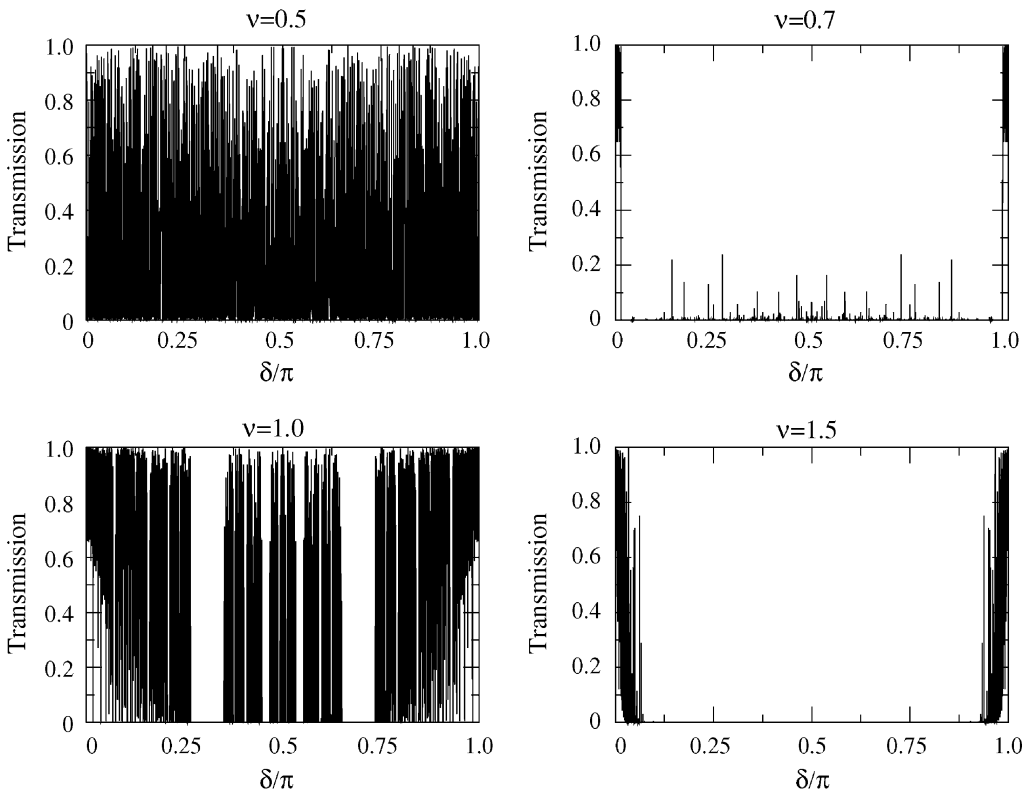

Fig. 1. Transmission spectra of multilayered structures with different values of ν and $N = 5 \times 10^3$ layers as a function of $\delta = \omega nd/c$ ($nd = n_A d_A = n_B d_B$). For $\nu > 1$, the structure is pseudo-random and the spectrum has narrow transmission peaks in the vicinity of the Bragg resonances. For $\nu < 1$, the transmission can be widely suppressed as ν increases. In the periodic incommensurate case ($\nu = 1$) the spectrum presents a self-similar structure of gaps typical of Fibonacci dielectric sequences.

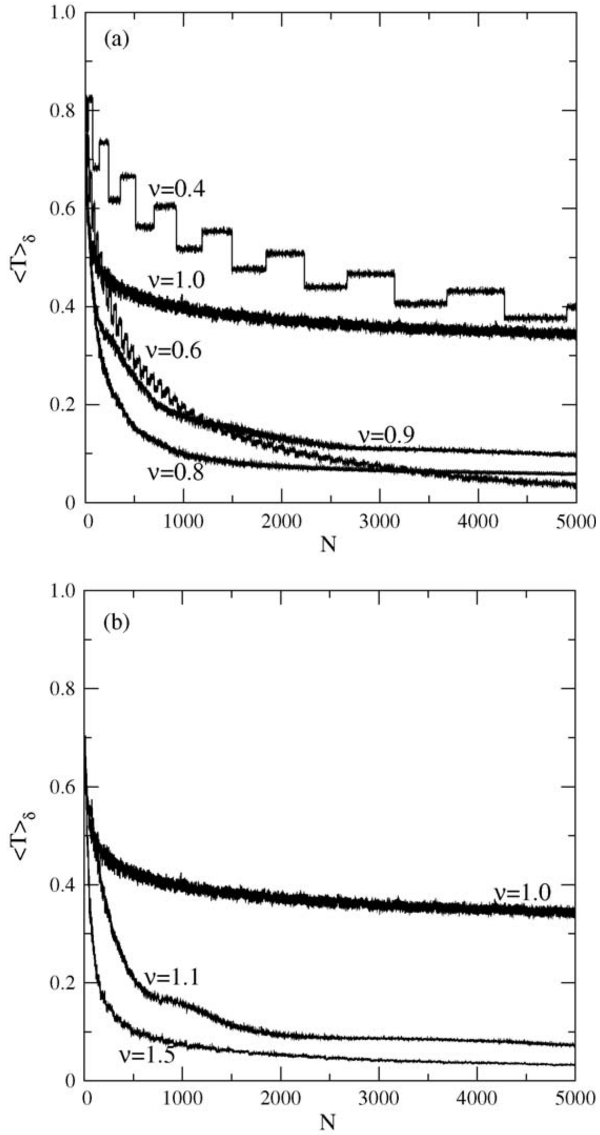

Fig. 2. Spectrally averaged transmission as a function of the number of layers N for different values of ν. (a) $\nu \leq 1.0$; (b) $\nu \geq 1.0$. For the periodic incommensurate case $\nu = 1$, the transmission displays a very weak size dependence. Notice that for smaller values of ν (roughly in the range $0.6 < \nu < 0.8$), the spectral average transmission of thick structures can become as small as that presented by pseudo-random structures with $\nu > 1$.

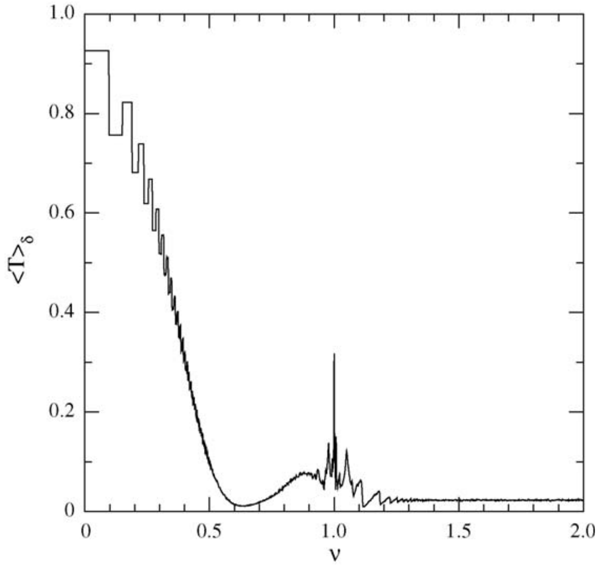

Fig. 3. Spectrally averaged transmission as a function of ν for a fixed sample thickness $N = 10^4$. The spectrally averaged transmission shows a local maximum at periodic incommensurate case $\nu = 1.0$ and a local minimum in the regime of $\nu < 1$. The plateau reached for $\nu > 1$ is a characteristic of the pseudo-random regime.

transmission spectra of multilayered structures with different ν values and $N = 5 \times 10^3$. As we used layers with the same optical length, the mode frequency is proportional to the phase change δ within each layer. For $\nu = 0.5$, the spectrum is dense with a large number of transmissing frequencies. This feature is due to the fact that the number of interfaces between distinct dielectric slabs is small in this regime. For $\nu = 1.0$, well-defined bandgaps are present that follow a self-similar pattern. This spectrum is typical of quasi-periodic structures with incommensurate sequences. For $\nu = 1.5$, the sequence presents a pseudo-random character and most of the modes become localized, except in the vicinity of the Bragg resonances. An interesting aspect is observed when we increase ν from $\nu = 0$ (uniform sequence) to $\nu = 1$ (quasi-periodic sequence). The number of transmitting modes changes non-monotonically in this region. This feature is represented by the case of $\nu = 0.7$ which exhibits just very few transmitting modes.

The above non-monotonic behavior of the transmission spectrum is described more quantitatively in Fig. 2, which shows the spectrally averaged transmission ($\langle T \rangle_\delta = (1/\pi) \int_0^\pi T(\delta) d\delta$) as a function of sample thickness N for different values of ν. The spectrally averaged transmission typically decreases as the number of layers increases. However, the size dependence is rather weak for the quasi-periodic case of $\nu = 1$, specially in the regime of large N. For $\nu > 1$, the decrease in the average transmission with the system size becomes faster, a feature associated with the pseudo-random character of the resulting sequence. The average transmission becomes readily ν-independent when ν further deviates from unit. On the other hand, in the regime of $\nu < 1$, the size dependence of the spectrally averaged transmission initially becomes stronger as ν increases, but reverses this trend as the quasi-periodic regime is approached.

A complimentary analysis of the influence of the aperiodicity exponent ν on the spectrally averaged transmission can be performed by plotting it as a

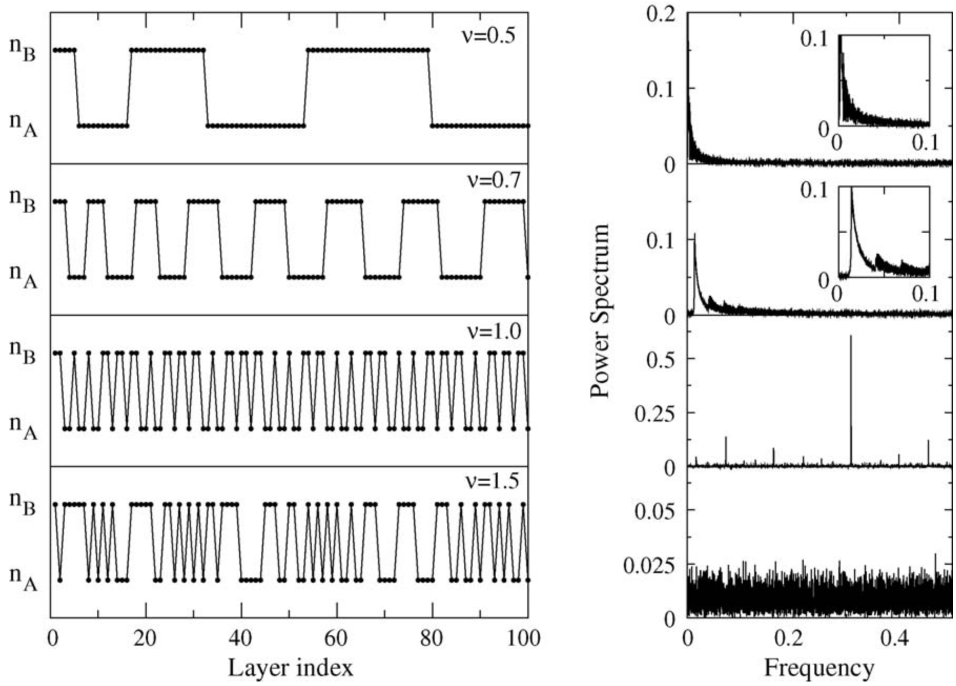

Fig. 4. Left panel: Typical sequences of refractive indices generated by the aperiodic modulated function. Right panel: the corresponding Fourier spectral densities of each refractive index sequence. For small values of ν, the spectral density displays a narrow peak around the average frequency of the sequence modulation. At $\nu = 1$ the structure follows a Fibonacci sequence which has a self-similar spectral density. For $\nu = 1.5$ the sequence is pseudo-random with a white-noise spectrum.

function of ν for a fixed sample thickness, as shown in Fig. 3 where we considered $N = 10^4$ layers. The averaged transmission depicts a peak at $\nu = 1$ with strong fluctuations in the vicinity of this quasi-periodicity condition. For large values of ν, the transmission is small, presenting weak fluctuations around a constant value. In the weakly aperiodic regime, it displays a well-defined minimum, signaling the crossover from the behavior of a uniform and a quasi-periodic structure.

The overall dependence of the averaged transmission on the aperiodicity exponent ν can be directly correlated to the spectral decomposition of the underlying sequence of layers. In Fig. 4, we plot typical sequences of refractive indices together with their Fourier spectral densities. Notice that, for small ν the Fourier spectrum displays a narrow peak at small frequencies, which reflects the long average wavelength of the refractive indices modulation. This peak becomes slightly broader as the number of layer interfaces increases. In the incommensurate case $\nu = 1$, the spectral density of the sequence assumes a self-similar pattern of delta-like peaks with a predominant frequency equals to $f = \alpha/2$, typical of a quasi-periodic Fibonacci sequence. In the regime of $\nu > 1$, the refractive index sequence resembles a fairly uncorrelated white-noise and the Fourier spectral density has no typical frequency.

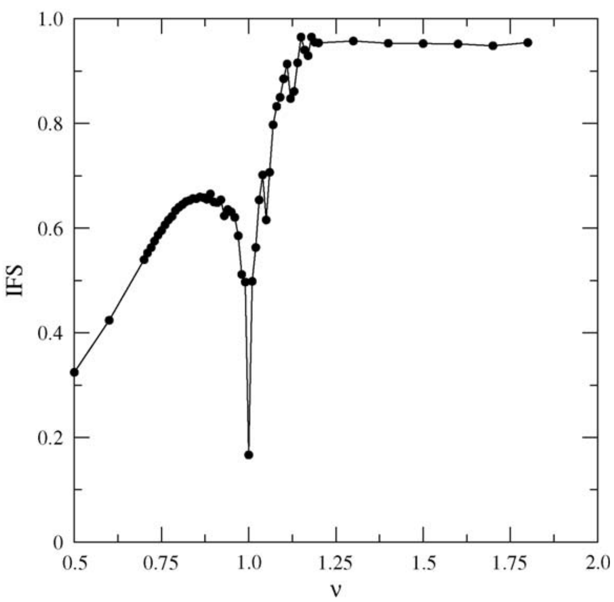

Fig. 5. Integrated Fourier spectrum (IFS) as a function of the aperiodicity exponent ν. The pronounced minimum at $\nu = 1$ signals the strong correlations presented by the underlying Fibonacci sequence of layers. The plateau for $\nu > 1$ reflects the uncorrelated nature of the dielectric layers sequence in this regime. The maximum at $\nu < 1$ is a consequence of opposite effects played by backscattering at interfaces and the emergence of resonant modes (see text).

The degree of correlation in the refractive index sequence can be quantified by computing the Integrated Fourier Spectrum (IFS). A noisy-like sequence shall have a large IFS while more regular structures will display a narrower Fourier spectrum and consequently a smaller IFS. The IFS of the refractive index sequences as a function of the aperiodic exponent ν is reported in Fig. 5. It exhibits a plateau at large values of ν pointing the irrelevance of the actual value of the aperiodicity exponent in the strongly pseudo-random regime. The deep minimum at $\nu = 1$ is associated with the quasi-periodic structure of the Fibonacci sequence for this particular case. In the weakly aperiodic regime, the IFS passes through a maximum as a result of the opposite roles played by the own inclusion of interfaces, which produces backscattering, and the approach to quasi-periodicity, which promotes the emergence of resonant modes. Notice that the overall behavior of the spectrally averaged transmission, presented in Fig. 3, indeed captures the essential features displayed by the IFS of the underlying structure. A strongly correlated sequence displays a narrow Fourier spectrum, thus leading to a small IFS and to a large averaged transmission.

4. Summary and conclusions

In this work, we investigated the transmission properties of 1D binary dielectric multilayered structures presenting an aperiodic modulation over the refractive index sequence. The aperiodicity degree was governed by a power-law exponent ν which controls the phase of a sinusoidal modulation. We considered the particular case for which the periodic case $\nu = 1$ corresponds to an incommensurate structure that resembles a Fibonacci sequence of refractive indices. Using a transfer matrix formalism, we computed the transmission spectrum along such stratified dielectric media. For $\nu > 1$, the resulting sequence becomes pseudo-random. In this regime, the Anderson localization of the electromagnetic modes leads to a wide gap in the transmission spectrum with narrow transmission bands centered at the Bragg resonances. In the regime of $\nu < 1$, we reported a non-monotonic dependence of the spectrally averaged transmission on the aperiodicity exponent α. Such unconventional behavior was associated with two opposite effects that take place in the presently investigated aperiodic structure. One effect is related to the backscattering on the interfaces between distinct dielectric slabs. This phenomenon leads to a reduction in the transmission when the number of interfaces starts to increase. However, when the number of interfaces becomes large enough, the quasi-periodic condition is approached which stabilizes resonant transmitting modes. As a consequence, the averaged transmission passes through a minimum which is smaller than the transmission in the strongly pseudo-random regime. A Fourier analysis showed that the trends presented by the transmission spectrum are directly associated with the correlations exhibited by the own refractive index sequence. Our results suggest that aperiodically modulated dielectric structures can be explored in the development of wide-band filters which can display an spectrally averaged transmission even smaller than the one presented by random structures.

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